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An Intelligible Software (CFA) Approach for Fiber-Reinforced Laminate Failure Analysis Including a Piecewise Representation of the Tsai-Wu Failure Criterion Using Excel and MatLab

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A Thesis Submitted to the Graduate Studies Office in Partial Fulfillment of the Requirements for the Degree of Master of Science in Aerospace Engineering

EMBRY-RIDDLE AERONAUTICAL UNIVERSITY

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This thesis was prepared under the direction of the candidate's thesis committee chairman, Dr. Yi Zhao, Department of Aerospace Engineering, and has been approved by the members of his thesis committee. It was submitted to the Aerospace Engineering Department and was accepted in partial fulfillment of the requirements for the degree of Master of Science in Aerospace Engineering.

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DEDICATION

This thesis is dedicated to my wife who has been an immense source of motivation and inspiration. Thank you for all of your support.

Also, this thesis is dedicated to my parents who have provided me with an enormous amount of guidance and support throughout my life. Thank you for providing me with copious opportunities and a superb education.

Finally, this thesis is dedicated to all those who strive for excellence devoid of recognition. Perseverance, vigilance, and conviction are all that is needed for success.

> "*Every day I remind myself that my inner and outer life are based on the labors of other men, living and dead, and that I must exert myself in order to give in the same measure as I have received and am still receiving.*" – Albert Einstein

> "*The democracy will cease to exist when you take away from those who are willing to work and give to those who would not.*" – Thomas Jefferson

> "*A society that will trade a little liberty for a little order will lose both, and deserve neither.*" – Thomas Jefferson

ACKNOWLEDGEMENTS

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ABSTRACT

Author: Ryan C. Schmidt

Title: An Intelligible Software (CFA) Approach for Fiber-Reinforced Laminate Failure Analysis Including a Piecewise Representation of the Tsai-Wu Failure Criterion Using Excel® and MatLab®

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Present generations rely heavily on the use of petroleum as their primary means of transportation. As the cost of petroleum continues to escalate, the need for lightweight structures for vehicle applications becomes more evident. The ability to engineer materials so that they possess desired application specific properties and characteristics has made tremendous progress in the past century. Consequently, the use of these composite materials for aircraft weight reduction has been investigated for decades.

The aerospace industry often uses composite materials to make a laminated composite structure where each constituent ply of the laminate has its own material properties. This anisotropic behavior of a fiber-reinforced laminate (FRL) is discussed methodically throughout this research. Moreover, the successful design of an FRL is dependent upon the accuracy of analyzing its structural limits. Therefore, the failure criteria used to specify an FRL's failure limits are significant.

Although useful formulaic methods have been developed for analyzing fiber-reinforced laminates, these calculations can be quite tedious when used in an iterative structural design process. Development of software that can conduct computer-aided laminate failure analysis can provide an indispensable tool for the design of fiber-reinforced laminate composites. Hence, this research focuses on the development of such software, CFA (Composite Failure Analysis). Even though fiber-reinforced laminate failure analysis is not a trivial topic, CFA was developed to provide a knowledgeable engineer with an intelligible software utensil for the design of fiber-reinforced laminates. Additionally, CFA's simplistic exploitation of Excel® and MatLab® make it an indispensable tool for engineering education instruction.

CFA exhibits an immense potential for the advancement of Embry-Riddle Aeronautical University's educational prowess in the field of composite materials. With its ability to perform fiber-reinforced laminate failure analysis using the universal engineering software platforms of Excel® and MatLab®, CFA provides the university with a novel capability for future composite materials research.

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NOMENCLATURE

$[A]$ – laminate extensional stiffness matrix

$[B]$ – laminate bending-stretching coupling matrix

$[C]$ – ply stiffness matrix

CFA – composite failure analysis software

CFRP – carbon-fiber reinforced polymer

CMC – ceramic matrix composite

$[D]$ – laminate flexural stiffness matrix

FRL – fiber-reinforced laminate

GFRP – glass-fiber reinforced polymer

$[M]$ – laminate force resultant matrix

MMC – metal matrix composite

$[N]$ – laminate force resultant matrix

PRM – piecewise representation method

$[Q]$ – ply reduced stiffness matrix

$[\bar{Q}]$ – ply transformed reduced stiffness matrix

$[S]$ – ply compliance matrix

σ – ply stress

$[T]$ – ply transformation matrix

τ – ply shear stress

[1-2-3] – principle material coordinate system

[x-y-z] – global laminate coordinate system

CHAPTER 1

Introduction

Present generations rely heavily on the use of petroleum as their primary means of transportation. As the cost of petroleum continues to escalate, the need for lightweight structures for vehicle applications becomes more evident. In the past, technological innovations in the aircraft industry were driven by the desire for higher performance. However, current advancements in aircraft technology are mostly due to the social economic requirements of lower cost, fuel efficient environmentally friendly operation. This represents a barrier for the aerospace industry which must be dealt with in a timely manner because the need for lightweight structures is of the utmost importance for the industry's prolonged longevity. Consequently, the ever increasing presence of composite material technology in today's aircraft is partially a result of this economic demand for improved fuel efficiency.

Before composites became more widely used for aircraft applications, aircraft structures were predominantly made from aircraft grade metal alloys. Although aircraft grade aluminum is a lightweight metal with respectable engineering properties, the development of lighter materials with equivalent or superior properties was a necessity. Technology quickly revealed that the use of composite materials could remedy this problem. Therefore, aircraft composite technology has become more widely used in today's aircraft industry. The search for composite materials that can replace their metal counterparts and maintain equivalent or superior engineering properties is at the forefront of research and development.

1.1 Research Introduction

The ability to engineer materials so that they possess desired application specific properties and characteristics has made tremendous progress in the past century. Advances in technology and economic incentive have pushed materials science to the forefront of research and development. The stanch appeal of the ability to control the properties of a material has lead researchers to focus on a branch of materials science known as composites.

A composite is defined as a material that has been engineered from two or more materials with appreciably different physical and/or chemical properties which remain visibly distinct. The use of composite materials for weight reduction has been the spotlight of the Aerospace Industry for decades. Although the standard construction and layout of aircraft has not changed for many years, the materials used to create them have become rather sophisticated. This is in part due to the economic benefits of aircraft weight reduction. With the aircraft industry's motto of 'weight reduction' driving the research and development of composite materials, advancements in this field are ever increasing. Because of this progress, the future of aircraft design is moving progressively toward that of a completely composite airframe structure. An example of the composite materials present in the next generation F-22 Raptor fighter aircraft is given in Figure 1.

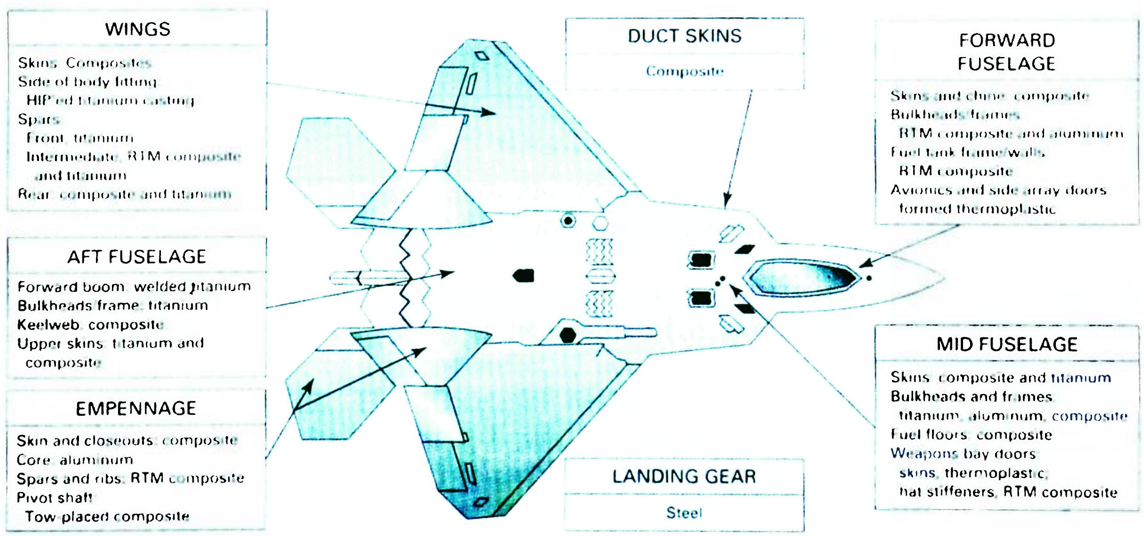

Figure 1: F-22 Raptor's use of composite materials [15].

The reason composites have become entwined with the aircraft industry is for the simple fact that they can be engineered to meet specified design requirements. Aircraft structural engineers no longer need to rely on saving weight solely by changing the type of metal alloy being used or by creating lightening cutouts in the structure itself; rather a composite material can be engineered for the specified structural application. What does this mean? This means that if a structure is subjected to a longitudinal and transverse tensile loading, then the composite material can be engineered to handle those specified loadings only; meaning the composite material structure will not be over designed as is often then case when using metal alloys. In the absence of strain hardening, metal alloys are usually considered to be isotropic. This means that the metal alloy's properties are homogeneous in all directions. This is not true in the case of composites. Composite materials are considered to be anisotropic. This means that a composite's engineering properties are directionally dependent. From a composite's anisotropic behavior, the significance of its application in the aerospace industry is evident. As an example, a chart of material strength-to-weight ratios versus temperature is provided in Figure 2.

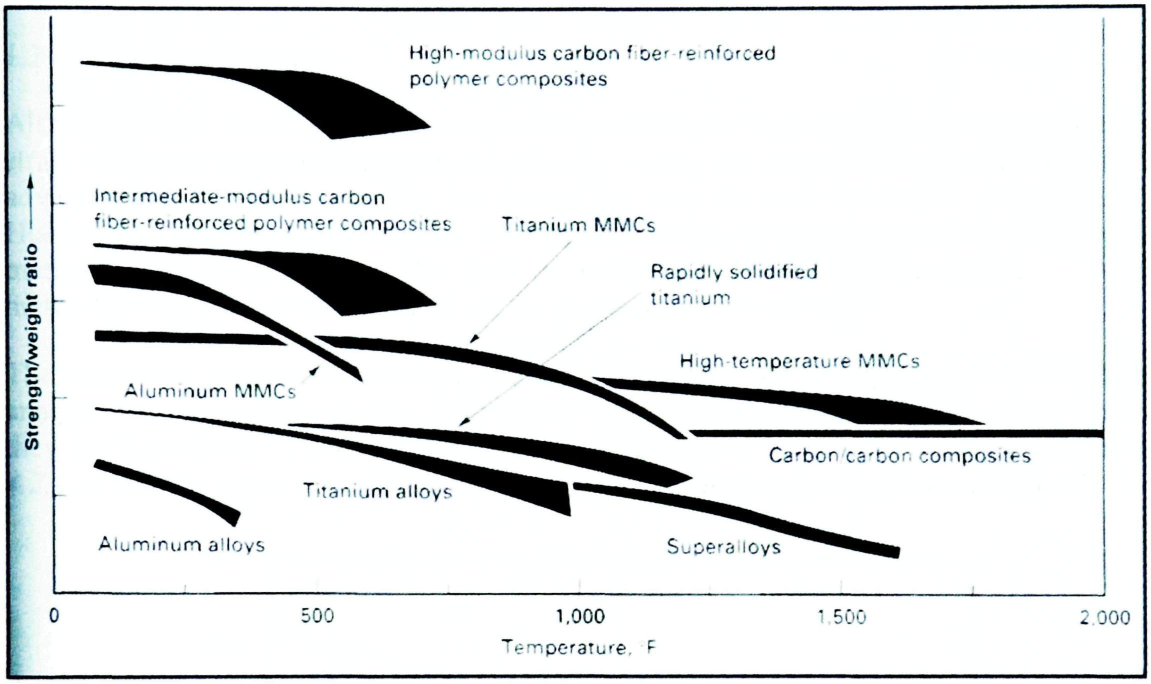

Figure 2: Example material strength-to-weight ratios versus temperature [15].

The Aerospace Industry often uses composite materials to make a laminated composite structure where each constituent ply of the laminate has its own material properties. Thus, the anisotropic behavior of a fiber-reinforced laminate (FRL) will be discussed in depth throughout this research. The successful design of an FRL is dependent upon the accuracy of analyzing its structural limits. Therefore, the criteria used to specify an FRL's failure limits are critical. Typically, an FRL is engineered through the use of a specific set of failure criteria that specify its failure limits. However, due to the unpredictable nature of the failure strength of analogous FRL specimens, it is helpful to use multiple failure criteria to conservatively determine its failure limits. Hence, the intention of this research is to create a computer program that is capable of performing fiber-reinforced laminate failure analysis using multiple failure criteria as well as to produce a piecewise representation of the Tsai-Wu failure criterion.

Furthermore, it is the intention of the author to conduct this research for the benefit of composite structural design. This research includes, but is not limited to, the creation of a composite failure analysis program called CFA. The CFA software is a combination of a sophisticated Excel® workbook and several MatLab® codes to produce an intelligible solution for fiber-reinforced laminate failure analysis. The codes written in MatLab® are used to provide users with a graphical representation of the failure analysis. Additionally, this research will present a piecewise representation of the Tsai-Wu Failure Criterion for multiple ply failure analysis.

Accordingly, the objectives of this research are twofold: 1) To provide a suitable analysis program (CFA) capable of performing fiber-reinforced laminate failure analysis, and 2) To present a piecewise representation of the Tsai-Wu Failure Criterion for multiple ply failure analysis.

1.2 Literature Survey

Although people have been using composite materials for thousands of years (i.e., mixing straw with mud to form strengthened bricks for walls), the relatively recent technological advancements of these materials has paved the way for new innovative technologies. Historically speaking, only recently have manufacturing technology and failure analysis methods been established in the field of composites. However, the failure analysis methods being used in this research are by no means novel concepts.

The maximum stress and maximum strain failure criteria are widely used for the failure analysis of composite materials. This is due to their inherent physical bases from which they are formulated [4]. These failure criteria are discussed in detail in *Sections 4.2.1* and *4.2.2*. A sample graph of a Maximum Stress Criterion application is given in Figure 3.

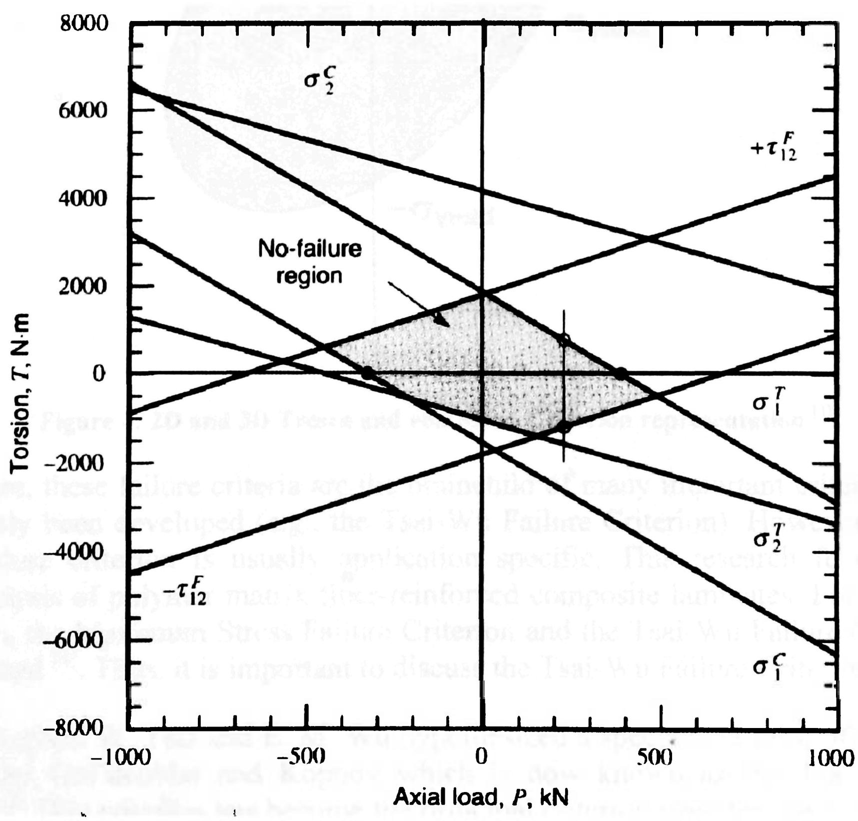

Figure 3: Sample of the application of Maximum Stress Criterion [4].

Although a number of failure criteria for the analysis of anisotropic materials have been postulated, it is important to recognize that most are simple variations of the maximum stress and maximum strain criteria discussed above [4]. However, there are a couple of well known criteria that are worth discussing.

In 1864, Henri Edouard Tresca pioneered the Maximum Shear Stress Criterion for the failure of materials which became later know as Tresca Theory [1]. Along with Tresca's efforts, in 1913, Richard Edler von Mises theorized the von Mises Yield Criterion [2] where the von Mises Yield Surface is that of a cylinder that bounds the Tresca Yield Surface given by Tresca Theory. This relationship can be seen in Figure 4.

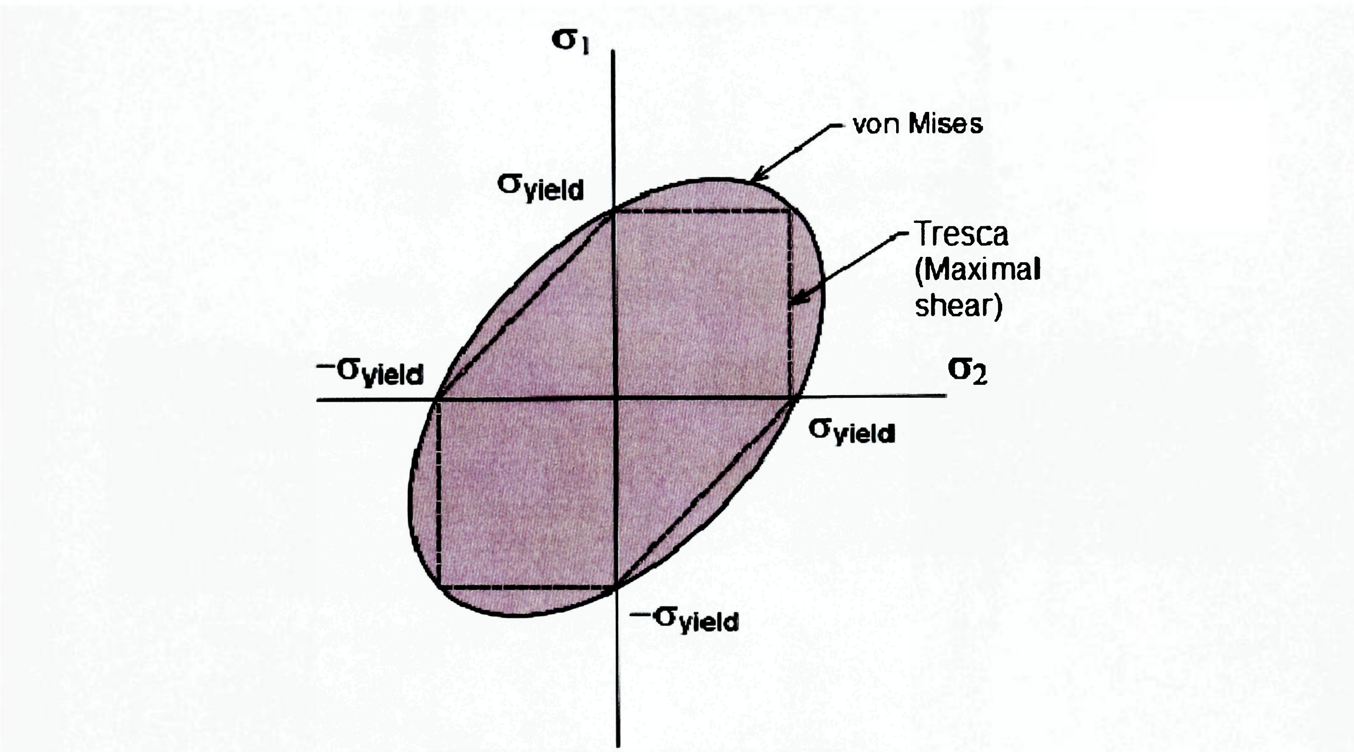

Figure 4: 2D and 3D Tresca and von Mises Criterion representation [3].

Furthermore, these failure criteria are the brainchild of many important criteria that have subsequently been developed (e.g., the Tsai-Wu Failure Criterion). However, the use of certain failure criterion is usually application specific. This research focuses on the failure analysis of polymer matrix fiber-reinforced composite laminates. For this type of application, the Maximum Stress Failure Criterion and the Tsai-Wu Failure Criterion are typically used [4]. Thus, it is important to discuss the Tsai-Wu Failure Criterion.

In 1971, Stephen W. Tsai and E. M. Wu hypothesized a specialized form of the criterion theorized by Gol'denblat and Kopnov which is now known as the Tsai-Wu Failure Criterion [5]. This criterion has become the principal criterion used for the failure analysis of polymer matrix fiber-reinforced composite laminates. A representation of this criterion is given in Figure 5.

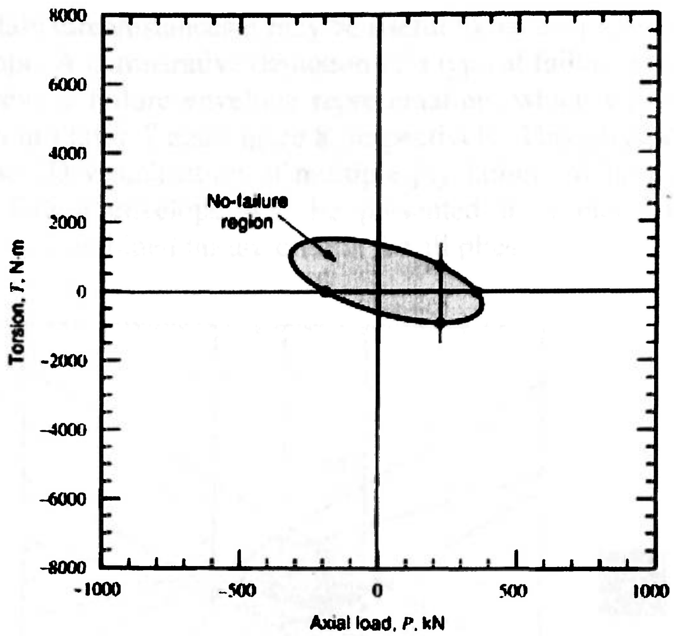

Figure 5: Sample of the application of Tsai-Wu Criterion [4].

What is the issue surrounding these failure criterion? In fact, this research does not claim that there is any downfall pertaining to their application. Rather, this research is focused on the concept of using multiple criteria to describe the failure behavior of a fiber-reinforced composite laminate as a whole. When the graph of the failure criteria incorporates multiple plies, the failure behavior that is obtained is referred to as a failure envelope. A representation of a failure envelope in which maximum stress and Tsai-Wu criteria were used can be seen in Figure 6.

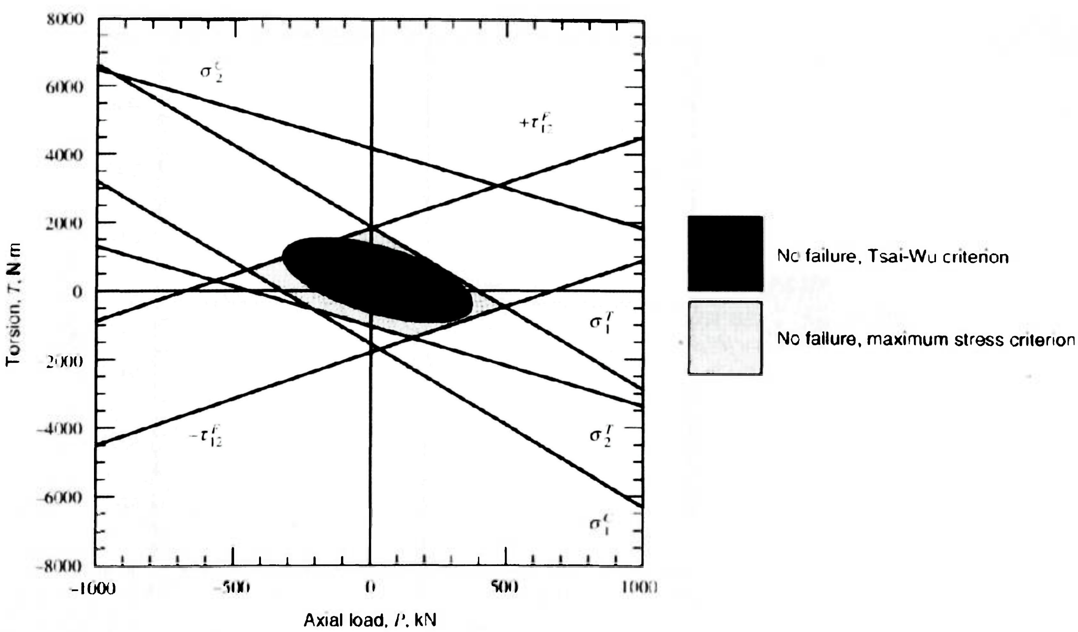

Figure 6: Sample of the application of Tsai-Wu and Maximum Stress Criterion [4].

Moreover, in certain circumstances it may be useful to have a piecewise representation of the failure envelope. A comparative depiction of a typical failure envelope representation to that of a piecewise failure envelope representation, which will be presented in this research, is given in Figure 7 and Figure 8, respectively. This piecewise representation is geared toward the 2D visualization of multiple ply failure. What does this mean? This means that the failure envelope will be presented in a piecewise solution that is representative of the combined failure criteria for all plies.

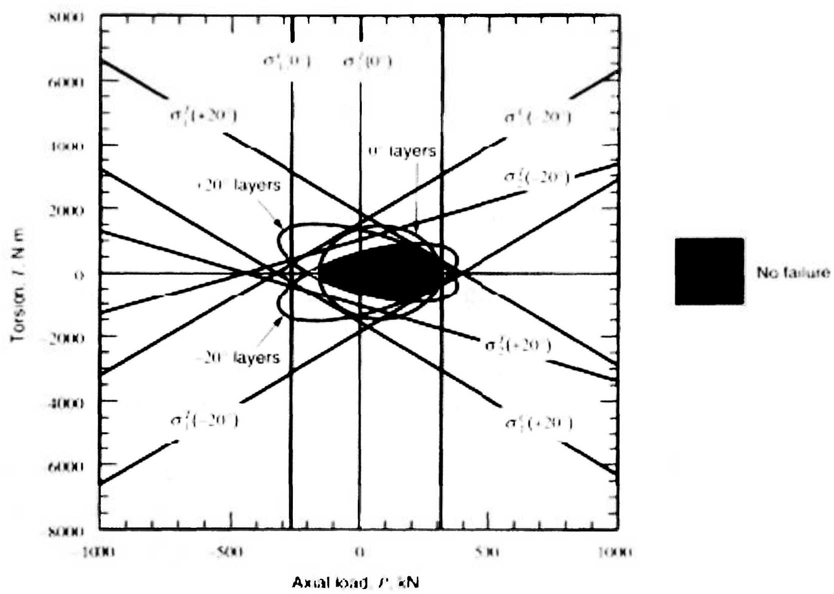

Figure 7: Sample of a typical multiple criteria failure envelope [4].

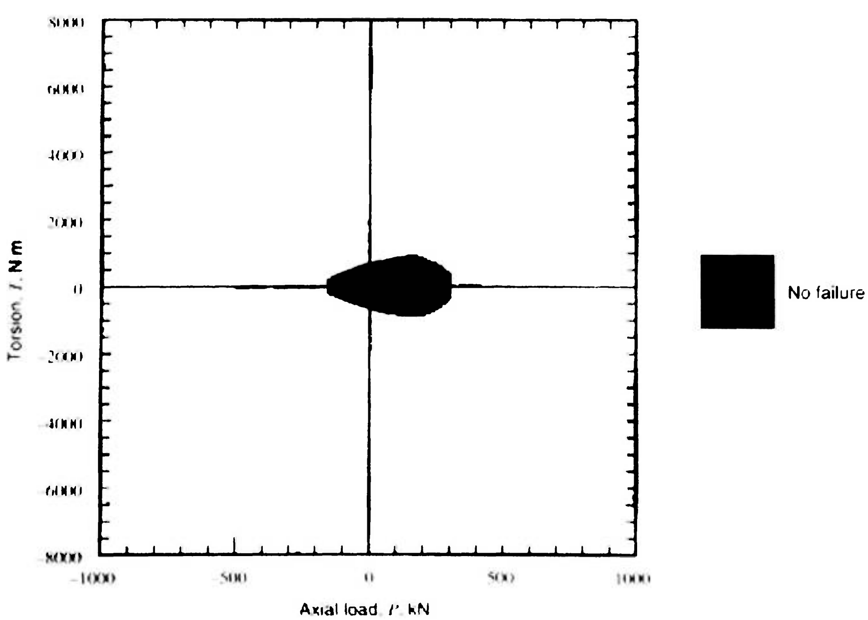

Figure 8: Sample of a piecewise failure envelope representation.

The addition of this approach for laminate failure analysis representation, to the field of composite laminate design, provides the ability for a multiple ply failure analysis, using failure envelopes as a representative description for the failure behavior of a fiber-reinforced composite laminate, to be modeled by a piecewise failure solution.

1.3 Research Limitations and Assumptions

Composite failure analysis and design is not a trivial subject. This research attempts to advance the field of composites through the creation of a sophisticated software program for fiber-reinforced laminate failure analysis with the ability to produce a piecewise representation of the Tsai-Wu failure criterion for multiple ply failure envelopes. However, there are few key assumptions and limitations that are pertinent to this research.

The limitations applicable to this research and used for simplification of failure analysis are as follows:

1. Free thermal strain effects have been neglected.
2. Free moisture strain effects have been neglected.
3. Hygrothermal effects have not been introduced.
4. Thermal stresses have not been introduced.

The assumptions applicable to this research and necessary for failure analysis using Classical Laminate Theory are as follows:

1. Orthotropic materials are assumed.
2. Maxwell reciprocal theorem is applied.
3. Plane-stress assumption is valid.
4. Kirchhoff hypothesis is valid.

A detailed understanding of the limitations and assumptions that have been discussed above is compulsory for understanding the methods used in this research. Knowledge of these topics as well as a basic understanding of the failure analysis of fiber-reinforced composite laminates is assumed throughout the remainder of this thesis document.

CHAPTER 2

Fiber-Reinforced Materials

Because the study of composite materials is so diverse, composites have many different areas of research and development. The study of composites ranges from the investigation of reinforced concrete to the design of carbon fiber reinforced polymers (CFRP). Having highly superior material properties makes it undoubtedly apparent as to why composite materials are used in such a diverse array of applications. Although other areas of composite research and development are significant in their own way, the field of fiber-reinforced composite materials is at the forefront of aviation materials research and development. Consequently, for the purpose of this research, it is imperative to have a detailed understanding of the mechanics of fiber-reinforced composite materials. However, since basic knowledge of the mechanics of fiber-reinforced composites is implied throughout this research document, the following fundamentals are only briefly discussed.

2.1 Introduction to Fiber-Reinforced Materials

Fiber-reinforced materials are by no means a novel idea. The concoction of a fibrous material and a bonding matrix has been used for many years. However, only relatively recently have they become engineered materials.

Although the design and engineering of materials can be profoundly complicated, the rewards of a successful design can be groundbreaking. For example, in the Aerospace Industry, fiber-reinforced composite materials have evolved into a way for engineers to significantly reduce the weight of an aircraft. This is often done by engineering composite materials with superior properties to replace their metal alloy counterparts.

By definition, a fiber-reinforced composite material is simply a material that is comprised of a fibrous material surrounded by a bonding matrix. For explicatory purposes, Table 1 illustrates the fiber/matrix composition of a few regularly used general fiber-reinforced composite materials.

Table 1: Constituent fiber/matrix materials for general fiber-reinforced composites.

Composite	Fiber	Matrix
Fiber-Glass	Glass	Epoxy Resin
Wood [8]	Cellulose	Lignin and Hemicellulose
Reinforced Concrete	Steel (Rebar)	Concrete
Rope [8]	Synthetic, Natural, or Metal	Air

The Aerospace Industry often uses fiber-reinforced composite materials that may be referred to as technical composites. An example of these aerospace materials and their fiber/matrix composition is given in Table 2.

Table 2: Constituent fiber/matrix materials for aerospace fiber-reinforced composites.

Composite	Fiber	Matrix
Carbon-Fiber Reinforced Polymer (CFRP)	Carbon	Polymer
Glass-Fiber Reinforced Polymer (GFRP)	Glass	Polymer
Metal Matrix Composite (MMC)	Synthetic, Natural, or Metal	Metal
Ceramic Matrix Composite (CMC)	Synthetic, Natural, or Metal	Ceramic

This research focuses on the failure analysis of technical aerospace composites, specifically continuous fiber-reinforced composites. For reference purposes, a compilation of a few of these materials and their respective properties has been arranged in *Appendix A.1*.

2.2 Principle Material Coordinate System

When studying fiber-reinforced composite materials, it is useful to utilize an orthogonal coordinate system that exploits one of its principle axes as the fiber direction of the composite. The principle material coordinate system, as it has become formally known as, represents a system that is defined in 1-2-3 space; where the 1-2-3 directions are representative of the material's fiber, width, and thickness directions, respectively. Therefore the 1-direction is known as the fiber direction while the other two directions are known as matrix directions corresponding to either the material's width or thickness. A representation of the principle material coordinate system can be seen in Figure 9.

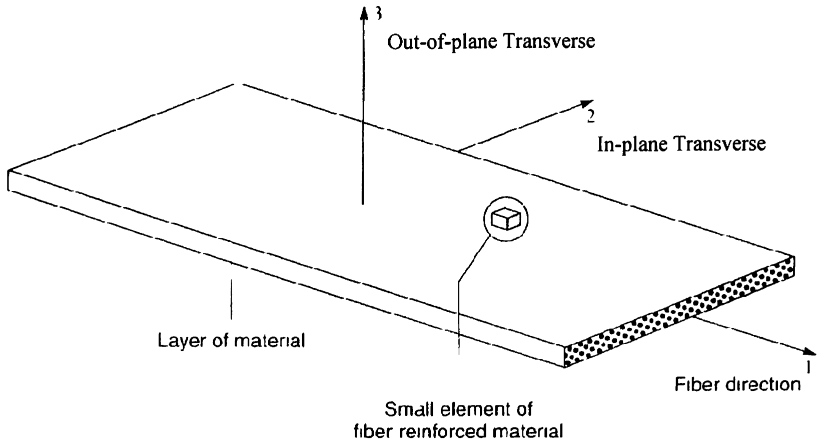

Figure 9: Principle material coordinate system representation [4].

The advantage to defining this coordinate system is that it allows for the engineering properties of the material to be expressed in terms of the 1-2-3 directions. In fact, when a composite material is tested in the laboratory, the engineering properties are determined with respect to its principle material coordinates. The usefulness of defining this coordinate system will become evident through the discussion of Classical Laminate Theory in *Section 3.6.*

2.3 Material Classification

A material that has homogeneous engineering properties in all directions is said to be an isotropic material. Therefore, it can be said that the engineering properties of an isotropic material are directionally independent. Typically, the isotropic material assumption is used throughout the study of metals. However, when a metal has been strain hardened, such as being subjected to a rolling process during machining, the metal can no longer be considered isotropic. The act of strain hardening a metal forces its engineering properties to become directionally dependent, specifically that in its rolling direction. When a material has directionally dependent engineering properties it is said to be anisotropic. Unsurprisingly, a composite material is classified as an anisotropic material. However, a few assumptions can be made for the purpose of simplifying analysis.

A material with different engineering properties in its three mutually perpendicular principle material directions, or planes of symmetry, is said to be an orthotropic material [4]. Naturally, this material classification can be attributed to fiber-reinforced composite laminates. Assumption of this material classification is implicit for the application of Classical Laminate Theory presented in *Section 3.6*. However, another assumption can be observed for further simplification.

A material is said to be transversely isotropic when it inherently has a plane of mutual equality in terms of its engineering properties. Logically, this assumption can be employed for a fiber-reinforced composite material because its 2-direction and 3-direction are constituently similar (assuming an isotropic matrix). Hence, both the 2-direction and the 3-direction are said to be in the 2-3 plane that is perpendicular to the material's fiber direction (1-direction) and therefore exhibit equal engineering properties. Assumption of this material classification is implicit when analyzing a fiber-reinforced composite material.

Accordingly, this research classifies a fiber-reinforced laminate as an orthotropic material with its constituent fiber-reinforced composite material layers being classified as transversely isotropic.

2.4 Stress-Strain Characteristics in the Principle Material Coordinate System

The failure analysis of fiber-reinforced laminates is directly dependent on the ability to accurately model its principle stresses. Therefore, for fiber-reinforced composite materials, it is helpful to represent a small elemental model of the material and its principal stresses as shown in Figure 10.

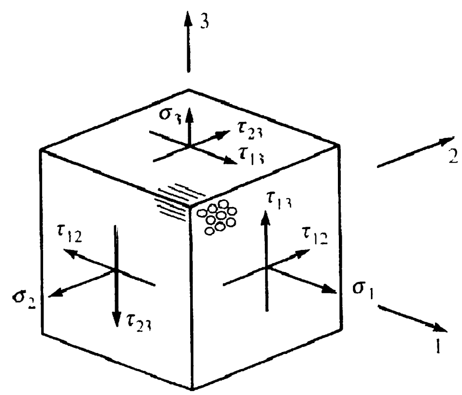

Figure 10: Stresses in a small element of a fiber-reinforced composite [4].

Furthermore, each application of stress illustrated in Figure 10 produces an elemental deformation that is specific to that of the applied stress. Deformations of an element due to the individual application of σ_1, σ_2, σ_3, τ_{23}, τ_{13}, and τ_{12} are given in Figure 11 through Figure 16, respectively.

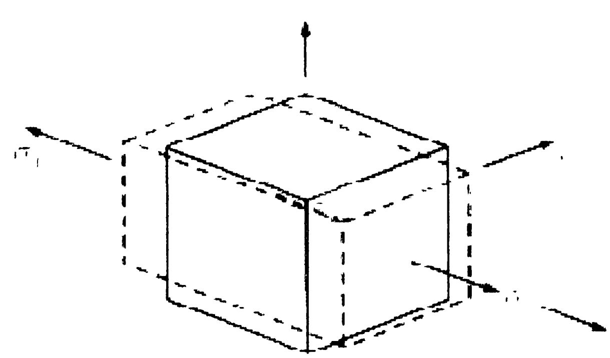

Figure 11: Elemental deformation due to σ_1 [4].

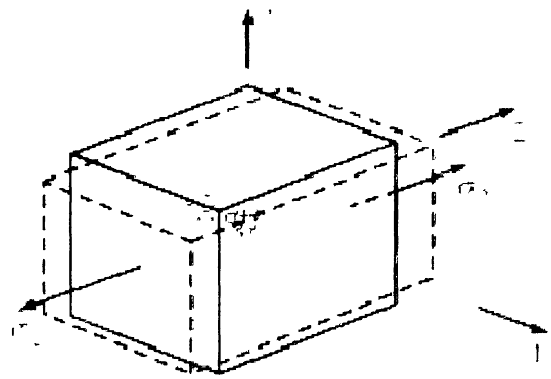

Figure 12: Elemental deformation due to σ_2 [4].

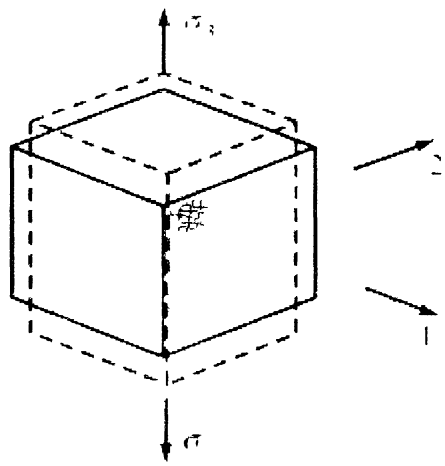

Figure 13: Elemental deformation due to σ_3 [4].

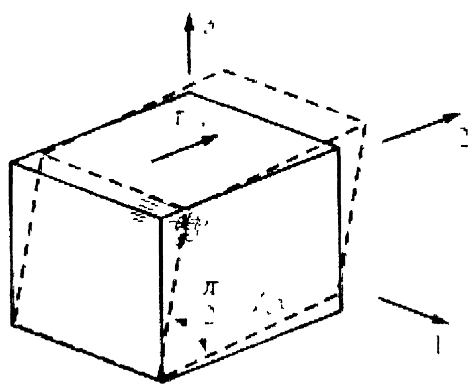

Figure 14: Elemental deformation due to τ_{23} [4].

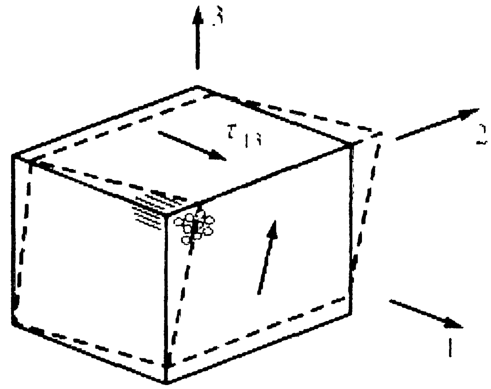

Figure 15: Elemental deformation due to τ_{13} [4].

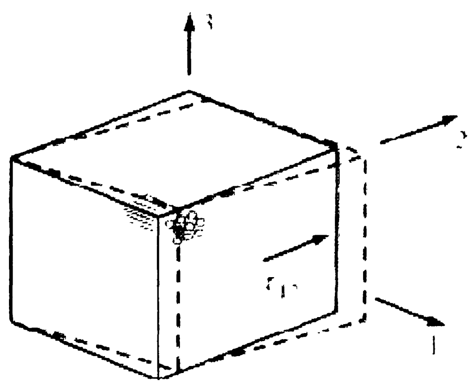

Figure 16: Elemental deformation due to τ_{12} [4].

With an understanding of the stresses that are present in a fiber-reinforced composite material, it is subsequently necessary to be able to quantify these stresses for purposes of failure analysis. Therefore, the 3D representation of the generalized Hooke's Law for anisotropic materials is used as a starting point for the development of the relationship between stress and strain in a fiber-reinforced composite material. This relationship is given by [9]

$$\sigma_i = C_{ij}\varepsilon_j \qquad i, j = 1, 2, 3, 4, 5, 6 \tag{2.1}$$

and

$$\varepsilon_i = S_{ij}\sigma_j \qquad i, j = 1, 2, 3, 4, 5, 6 \tag{2.2}$$

where the compliances, S_{ij}, are given as the inverse of the stiffnesses, C_{ij}. Furthermore, these stress strain relations can be written as matrices of the form

$$
\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} \tag{2.3}
$$

and

$$
\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} \tag{2.4}
$$

For illustrative purposes, a representation of the physical significance of the anisotropic stress-strain relation, related through the compliance [S] matrix, is given in Figure 17.

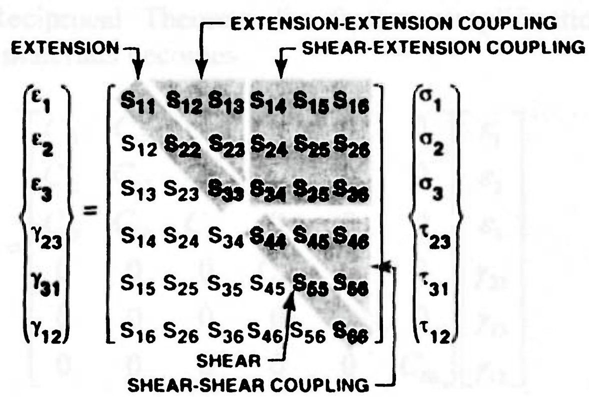

Figure 17: Physical significance of the anisotropic stress-strain relation [9].

Additionally, with respect to fiber-reinforced composite materials, when the orthotropic material assumption is applied (see *Reference 4*), the stress-strain relation reduces to

$$
\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} \tag{2.5}
$$

and

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} \tag{2.6}$$

Now, the Maxwell Reciprocal Theorem states that

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \tag{2.7}$$

and therefore it can be shown that

$$C_{ij} = C_{ji} \qquad S_{ij} = S_{ji} \tag{2.8}$$

Details regarding the Maxwell Reciprocal Theorem can be found in *Reference 4*.

Using the Maxwell Reciprocal Theorem for further simplification, the stress-strain relation for orthotropic materials becomes

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} \tag{2.9}$$

and

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} \tag{2.10}$$

The orthotropic stress-strain relation is subsequently determined using the stiffness [C] matrix in terms of the compliances [S] matrix given by

$$
\begin{aligned}
C_{11} &= \frac{S_{22}S_{33}-S_{23}S_{23}}{S} & C_{12} &= \frac{S_{13}S_{23}-S_{12}S_{33}}{S} \\
C_{22} &= \frac{S_{33}S_{11}-S_{13}S_{13}}{S} & C_{13} &= \frac{S_{12}S_{23}-S_{13}S_{22}}{S} \\
C_{33} &= \frac{S_{11}S_{22}-S_{12}S_{12}}{S} & C_{23} &= \frac{S_{12}S_{13}-S_{23}S_{11}}{S} \\
C_{44} &= \frac{1}{S_{44}} \qquad C_{55} = \frac{1}{S_{55}} & C_{66} &= \frac{1}{S_{66}}
\end{aligned}
\tag{2.11}
$$

where

$$
S = S_{11}S_{22}S_{33} - S_{11}S_{23}S_{23} - S_{22}S_{13}S_{13} - S_{33}S_{12}S_{12} + 2S_{12}S_{23}S_{13} \tag{2.12}
$$

and

$$
\begin{aligned}
S_{11} &= \frac{1}{E_1} & S_{12} &= -\frac{\nu_{12}}{E_1} & S_{13} &= -\frac{\nu_{13}}{E_1} \\
S_{22} &= \frac{1}{E_2} & S_{23} &= -\frac{\nu_{23}}{E_2} & S_{33} &= \frac{1}{E_3} \\
S_{44} &= \frac{1}{G_{23}} & S_{55} &= \frac{1}{G_{13}} & S_{66} &= \frac{1}{G_{12}}
\end{aligned}
\tag{2.13}
$$

Furthermore, if a material is transversely isotropic, as discussed in *Section 2.3*, then it can be shown that

$$
\begin{aligned}
E_2 &= E_3 & \nu_{12} &= \nu_{13} \\
G_{12} &= G_{13} & G_{23} &= \frac{E_2}{2(1+\nu_{23})}
\end{aligned}
\tag{2.14}
$$

and therefore the stress-strain relation becomes

$$
\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix}
=
\begin{bmatrix}
C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{12} & C_{23} & C_{22} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{55}
\end{bmatrix}
\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix}
\tag{2.15}
$$

and

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{12} & S_{23} & S_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{55} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} \tag{2.16}$$

The transversely isotropic stress-strain relation is subsequently determined using the stiffness [C] matrix in terms of the compliances [S] matrix given by

$$\begin{aligned} C_{11} &= \frac{S_{22}S_{22} - S_{23}S_{23}}{S} & C_{12} &= \frac{S_{12}S_{23} - S_{12}S_{22}}{S} & C_{44} &= \frac{1}{S_{44}} \\ C_{22} &= \frac{S_{22}S_{11} - S_{12}S_{12}}{S} & C_{23} &= \frac{S_{12}S_{12} - S_{23}S_{11}}{S} & C_{55} &= \frac{1}{S_{55}} \end{aligned} \tag{2.17}$$

where S is determined by

$$S = S_{11}S_{22}S_{22} - S_{11}S_{23}S_{23} - S_{22}S_{12}S_{12} - S_{22}S_{12}S_{12} + 2S_{12}S_{23}S_{12} \tag{2.18}$$

and

$$\begin{aligned} S_{11} &= \frac{1}{E_1} & S_{12} &= -\frac{\nu_{12}}{E_1} & S_{22} &= \frac{1}{E_2} \\ S_{23} &= -\frac{\nu_{23}}{E_2} & S_{44} &= \frac{1}{G_{23}} = \frac{2(1+\nu_{23})}{E_2} & S_{55} &= \frac{1}{G_{12}} \end{aligned} \tag{2.19}$$

Accordingly, with respect to this research, the aforementioned relationships between stress and strain, developed in the principle material coordinate system, for both orthotropic and transversely isotropic fiber-reinforced composite materials will be used in subsequent failure analysis.

2.5 Plane-Stress Assumption

Frequently, the plane-stress assumption is employed for the development of the mechanics of fiber-reinforced materials [4]. For the purposes of this research, the plane-stress assumption stipulates that, for laminates, the stresses in the plane of the laminate are much larger than the stresses perpendicular to the laminate plane [4]. Therefore, with this assumption, all stress components that are perpendicular to the laminate plane are subsequently negligible. The stress-strain relation for orthotropic materials evaluated at $\sigma_3 = \tau_{23} = \tau_{13} = 0$ becomes

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ 0 \\ 0 \\ 0 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} \tag{2.20}$$

and

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ 0 \\ 0 \\ 0 \\ \tau_{12} \end{bmatrix} \tag{2.21}$$

From the above stress-strain relations, it is evident that $\gamma_{23} = \gamma_{13} = 0$. However, although it is clear that $\sigma_3 = 0$, it is not as apparent that $\varepsilon_3 \neq 0$. Through matrix computation it can be determined that

$$\varepsilon_3 = -\frac{C_{13}}{C_{33}}\varepsilon_1 - \frac{C_{23}}{C_{33}}\varepsilon_2 \tag{2.22}$$

As long as the fact that $\varepsilon_3 \neq 0$ is not overlooked in the design process, the stress-strain relation can be reduced to

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} \tag{2.23}$$

Because $\varepsilon_3 \neq 0$, it is necessary to perform a few matrix computations in order to express the stress-strain relation in terms of the stiffness [C] matrix. So, substituting for ε_3 gives

$$\begin{aligned} \sigma_1 &= C_{11}\varepsilon_1 + C_{12}\varepsilon_2 + C_{13}\left(-\frac{C_{13}}{C_{33}}\varepsilon_1 - \frac{C_{23}}{C_{33}}\varepsilon_2\right) \\ \sigma_2 &= C_{12}\varepsilon_1 + C_{22}\varepsilon_2 + C_{23}\left(-\frac{C_{13}}{C_{33}}\varepsilon_1 - \frac{C_{23}}{C_{33}}\varepsilon_2\right) \end{aligned} \tag{2.24}$$

which can be written as

$$\sigma_1 = \left(C_{11} - \frac{C_{13}^2}{C_{33}} \right) \varepsilon_1 + \left(C_{12} - \frac{C_{13}C_{23}}{C_{33}} \right) \varepsilon_2$$
$$\sigma_2 = \left(C_{12} - \frac{C_{13}C_{23}}{C_{33}} \right) \varepsilon_1 + \left(C_{22} - \frac{C_{23}^2}{C_{33}} \right) \varepsilon_2 \tag{2.25}$$

It can then be determined that the stress-strain relation for the state of plane-stress of a fiber-reinforced 2-3 orthotropic composite material is given by

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{2.26}$$

where the reduced stiffness [Q] matrix is evaluated using

$$\begin{aligned} Q_{11} &= C_{11} - \frac{C_{13}^2}{C_{33}} = \frac{E_1}{1-\nu_{12}\nu_{21}} \\ Q_{12} &= C_{12} - \frac{C_{13}C_{23}}{C_{33}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} \\ Q_{22} &= C_{22} - \frac{C_{23}^2}{C_{33}} = \frac{E_2}{1-\nu_{12}\nu_{21}} \\ Q_{66} &= C_{66} = G_{12} \end{aligned} \tag{2.27}$$

Accordingly, these representations of the stress-strain relations for the state of plane-stress are used later in this research. However, using the plane-stress assumption for analysis requires knowledge of a few issues that are commonly overlooked.

When it comes to utilizing the plane-stress assumption, the two major drawbacks that are directly associated with its use are [4]:

1. Typically, there is no attempt to calculate the out-of-plane stress components that were equated to zero. Often, these stresses are forgotten.
2. In error, it is often assumed that since the stress in the 3-direction (σ_3) is equated to zero then the strain associated with this stress (ε_3) is also zero.

With regards to the former, the out-of-plane stress components are assumed to be a great deal smaller than that of the in-plane stress components. Equating these components to zero is a valid assumption only if the interlaminar stresses are sustainable. Therefore, it is obvious that this assumption can only be used if it is known to the designer that the material is indeed strong enough to resist the neglected interlaminar stress components. In certain situations where delamination of the material is a relevant concern, ignorance of these out-of-plane stress components can be a drastic oversight.

With regards to the later, the derivation and application of the plane-stress assumption presumes that the out-of-plane stress components are subsequently equal to zero. With this assumption, the equivalent principle stress in the 3-direction (σ_3) is equated to zero. Furthermore, through the simplification of Hooke's Law for orthotropic materials, it is actually proven that even though $\sigma_3 = 0$, $\varepsilon_3 \neq 0$. Therefore, it is necessary to make sure that the material can withstand this interlaminar strain.

For the purpose of this research, the plane-stress assumption is used for the failure analysis of fiber-reinforced laminates. Consequently, it is assumed that the out-of-plane stress components are sustainable by the laminate and therefore the calculation of ε_3 unnecessary. Although, for design purposes, it is necessary to investigate the interlaminar stress component relations of the material, however, further consideration of this subject is not necessary for the purposes of this research.

CHAPTER 3

Fiber-Reinforced Laminates

The use of fiber-reinforced materials in the Aerospace Industry is becoming exponential. However, the manner in which these materials are being used is what makes them unique in the materials engineering world.

Fiber-reinforced materials are mostly used to produce composite laminates. These laminates are made of multiple layers of fiber-reinforced materials called plies. The use of laminates in the Aerospace Industry is ever increasing. They are used for fuselage panels, ingress and egress structures, wing surfaces, and even wing spars.

Laminates can be engineered to perform diverse roles within an aircraft's structure. They can be used as structural members in major load bearing components of the aircraft or simply as lightweight structures within the aircraft that were previously over-designed through the use of typical aluminum alloys. Because the applications of fiber-reinforced laminates can be so diverse, the Aerospace Industry has turned to these unconventional materials as their vehicle for reducing aircraft weight.

3.1 Introduction to Fiber-Reinforced Laminates

Fiber-reinforced laminates have become the predominant composite material for use in the Aerospace Industry. Although fiber-reinforced materials have previously been discussed, it is necessary to introduce the concept of a fiber-reinforced material in the form of a laminate. Therefore, it is fundamental to define a laminate as a structure that incorporates multiple layers of distinctly separate materials. These materials do not have to be different in composition or orientation. They are only required to be distinctly separate within the laminate.

The purpose of creating a laminate with distinctly separate material layers is to design a material that exhibits superior mechanical properties in the directions needed to sustain its applicable loadings. To do this, fibrous materials are often used as they can be engineered to provide suitable mechanical properties for their desired application. Using fiber-reinforced materials to create a laminate allows for the designer to create a structure that can be a concoction of different plies and/or material types in order to produce a lightweight and viable structure.

The concepts behind the analysis and design of fiber-reinforced laminates will be explored throughout this chapter. For illustrative purposes, a representation of the concepts that will be introduced is given in Figure 18.

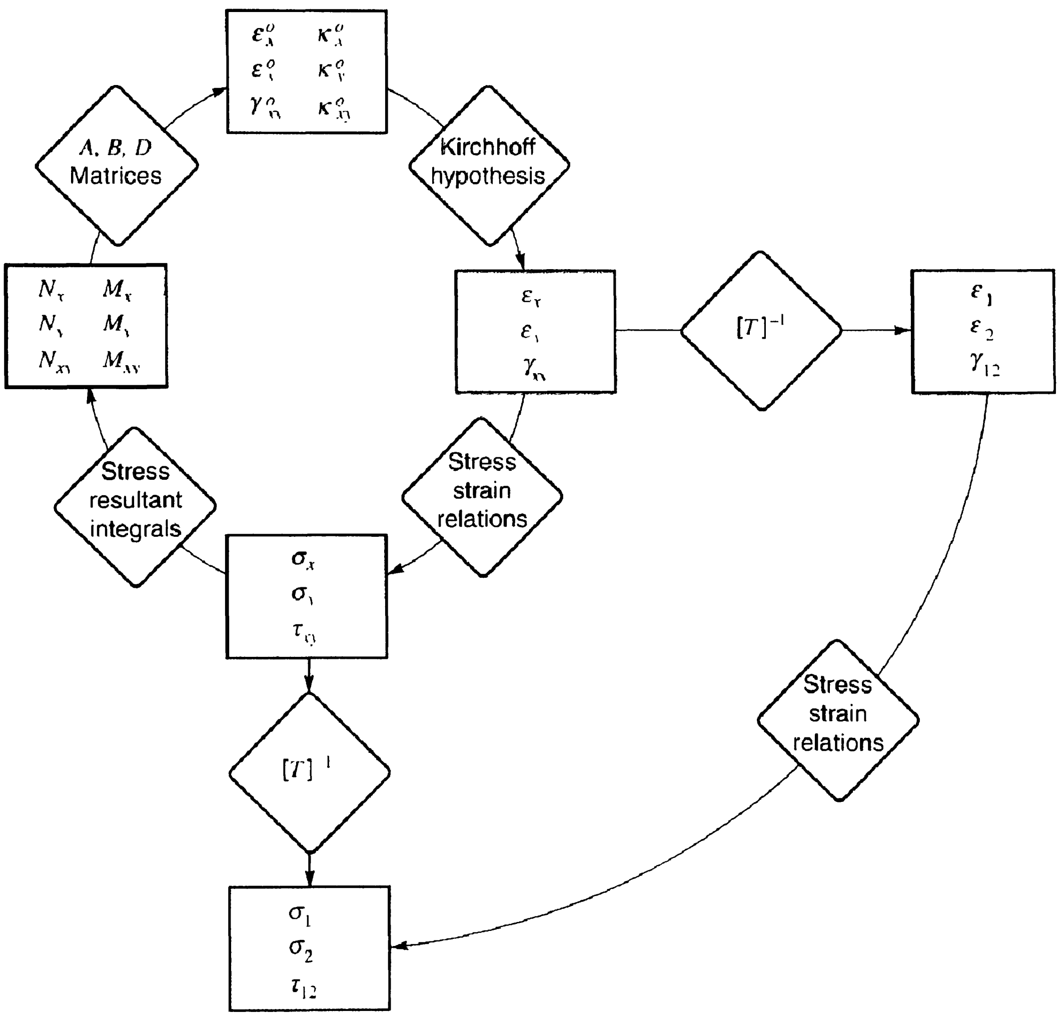

Figure 18: Illustration of fiber-reinforced laminate analysis concepts [4].

3.2 Global Laminate Coordinate System

Much like the principle material coordinate system that was established in *Section 2.2*, the global laminate coordinate system is introduced as an arbitrary coordinate system that defines the x-y-z space of a laminate. Although this coordinate system is arbitrary and can be selected by the designer, it is typical that the coordinate system is selected in such a manner that the laminate's length, width, and thickness dimensions corresponds to the x, y, and z directions, respectively. A comparison of the principle material (1-2-3) and the global laminate (x-y-z) coordinate systems can be seen in Figure 19.

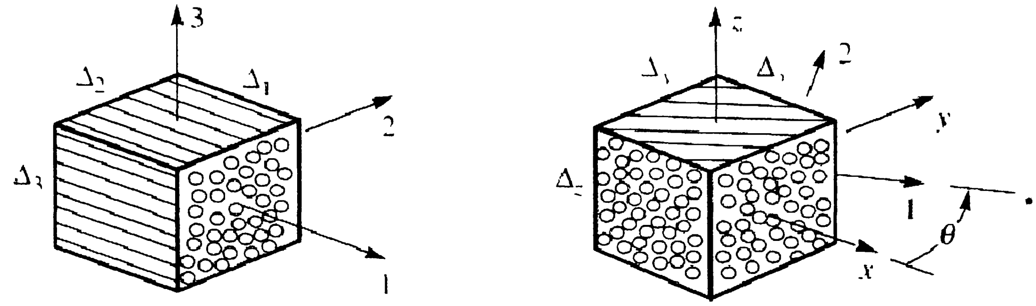

Figure 19: Principle 1-2-3 material and global laminate x-y-z coordinate systems [4].

For purposes of this research, it is necessary to note the laminate sign convention that has been adopted for subsequent analysis. This sign convention is illustrated in Figure 20, with the directional representations of x, y, and z being positive.

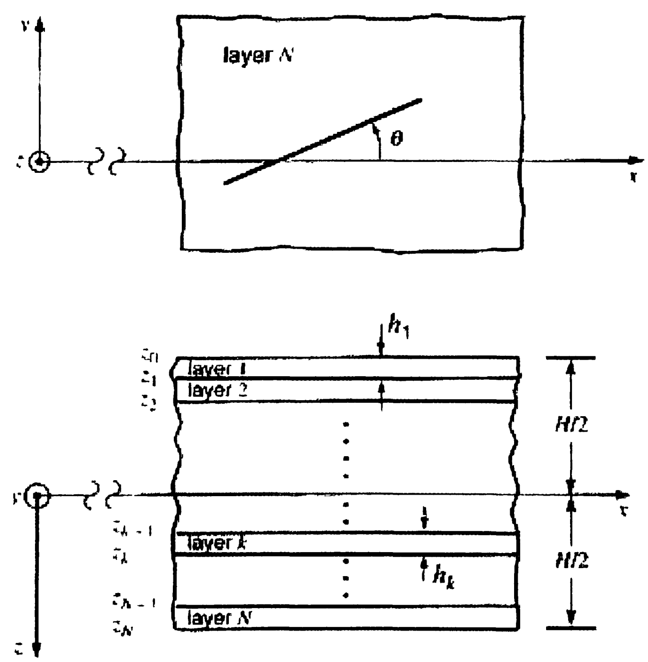

Figure 20: Sign convention for global coordinate system [4].

3.3 Stress-Strain Characteristics in the Global Laminate Coordinate System

The stress-strain characteristics of a laminate were previously discussed for the principle material coordinate system in *Section 2.4*. However, how do you analyze a laminate with multiple plies that are all at different ply angles?

In order to analyze a laminate, it is necessary to define a global laminate coordinate system. This would suggest that each ply in the laminate would have its own relative principle material coordinate system that can then be related to the global laminate coordinate system of the laminate. In order to accomplish this, the relations between the ply's principle material coordinate system and the laminate's global laminate coordinate system must be developed.

Figure 21 illustrates the stresses of an elemental section of a laminate with the directional representations of the stresses being positive.

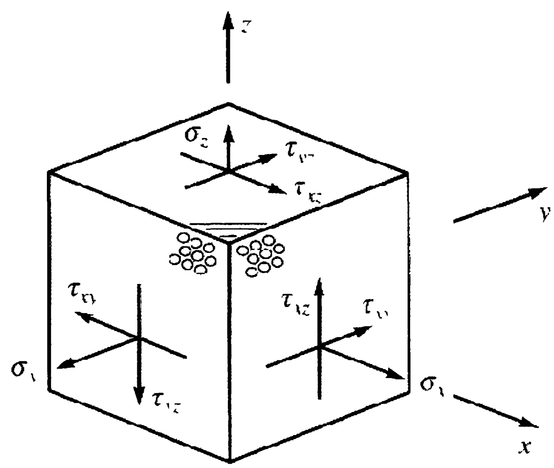

Figure 21: Global stresses in an elemental section of a laminate [4].

For a state of plane-stress, the relations between the principle and global strains are

$$
\begin{aligned}
\varepsilon_1 &= \cos^2\theta\varepsilon_x + \sin^2\theta\varepsilon_y + 2\sin\theta\cos\theta\frac{1}{2}\gamma_{xy} \\
\varepsilon_2 &= \sin^2\theta\varepsilon_x + \cos^2\theta\varepsilon_y - 2\sin\theta\cos\theta\frac{1}{2}\gamma_{xy} \\
\frac{1}{2}\gamma_{12} &= -\sin\theta\cos\theta\varepsilon_x + \sin\theta\cos\theta\varepsilon_y + \left(\cos^2\theta - \sin^2\theta\right)\frac{1}{2}\gamma_{xy}
\end{aligned}
\tag{3.1}
$$

However, these relations are more commonly expressed in terms of matrices as

$$
\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ (1/2)\gamma_{12} \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ (1/2)\gamma_{xy} \end{Bmatrix} \tag{3.2}
$$

where T is the transformation matrix given by

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \tag{3.3}$$

and $m = \cos\theta$ and $n = \sin\theta$.

Since $\varepsilon_3 \neq 0$, developed in *Section 2.5*, the preceding transformation to the global laminate coordinate system will result in $\varepsilon_z \neq 0$. Therefore, ε_z can be written as

$$\varepsilon_z = S_{13}\sigma_1 + S_{23}\sigma_2 \tag{3.4}$$

As discussed previously, when designing fiber-reinforced composites using the plane-stress assumption, it is vital to remember that $\varepsilon_3 \neq 0$ and $\varepsilon_z \neq 0$. The laminate must be able to overcome these interlaminar stresses or delamination can occur. For the purposes of this research, it is assumed that the interlaminar stresses are sustainable.

Furthermore, a transformed reduced compliance matrix can be obtained using these relations which can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{3.5}$$

where

$$\begin{aligned} \bar{S}_{11} &= S_{11}m^4 + (2S_{12} + S_{66})n^2m^2 + S_{22}n^4 \\ \bar{S}_{12} &= (S_{11} + S_{22} - S_{66})n^2m^2 + S_{12}(n^4 + m^4) \\ \bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66})nm^3 - (2S_{22} - 2S_{12} - S_{66})n^3m \\ \bar{S}_{22} &= S_{11}n^4 + (2S_{12} + S_{66})n^2m^2 + S_{22}m^4 \\ \bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66})n^3m - (2S_{22} - 2S_{12} - S_{66})nm^3 \\ \bar{S}_{66} &= 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66})n^2m^2 + S_{66}(n^4 + m^4) \end{aligned} \tag{3.6}$$

These relations can also be used to obtain the transformed reduced stiffness matrix

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.7}$$

where

$$
\begin{aligned}
\bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}n^4 \\
\bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})n^2m^2 + Q_{12}(n^4 + m^4) \\
\bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m \\
\bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})n^2m^2 + Q_{22}m^4 \\
\bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})n^3m + (Q_{12} - Q_{22} + 2Q_{66})nm^3 \\
\bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)
\end{aligned}
\tag{3.8}
$$

The transformed reduced compliance and stiffness matrices, for the state of plane-stress, are essential for the application of Classical Laminate Theory in *Section 3.6.*

3.4 Engineering Properties in the Global Laminate Coordinate System

Now that the global laminate coordinate system has been defined, it is constructive to determine a ply's engineering properties in its global laminate coordinate system. Through the use of Hooke's Law and the transformation relationships developed in *Section 3.3*, the engineering properties for a ply in the global laminate coordinate system are given as

$$
\begin{aligned}
E_x &= \frac{E_1}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)n^2m^2 + \frac{E_1}{E_2}n^4} \\
\nu_{xy} &= \frac{\nu_{12}(n^4 + m^4) - \left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right)n^2m^2}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)n^2m^2 + \frac{E_1}{E_2}n^4} \\
E_y &= \frac{E_2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right)n^2m^2 + \frac{E_2}{E_1}n^4} \\
\nu_{xy} &= \frac{\nu_{21}(n^4 + m^4) - \left(1 + \frac{E_2}{E_1} - \frac{E_2}{G_{12}}\right)n^2m^2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right)n^2m^2 + \frac{E_2}{E_1}n^4} \\
G_{xy} &= \frac{G_{12}}{n^4 + m^4 + 2\left(2\frac{G_{12}}{E_1}(1 + 2\nu_{12}) + 2\frac{G_{12}}{E_2} - 1\right)n^2m^2}
\end{aligned}
\tag{3.9}
$$

3.5 Laminate Nomenclature

Now that most of the essential stress-strain relationships have been introduced for plies in the principle material and global laminate coordinate systems, further discussion of fiber-reinforced laminates nomenclature is required.

A simple designation has been developed for the ease of naming fiber-reinforced laminates. The designation starts with the first layer of the laminate and ends with the last. With reference to Figure 20 for the sign convention and stacking sequence of a fiber-reinforced laminate, a trivial designation process can be applied to any laminate.

For example, take a laminate with a designation of [0/30/45]. This laminate consists of 3 plies with layer 1 at a ply angle of 0°, layer 2 at a ply angle of 30°, and layer 3 at a ply angle of 45°. A symmetric ply is denoted with a subscript *S*, whereas the subscript *T* is used to denote the total laminate. Therefore, a [0/90/30/30/90/0] laminate designation is equivalent to a $[0/90/30]_S$ designation because the laminate is symmetric. A few examples of laminate designations are given in Figure 22 and Figure 23.

Layer Sequence	Laminate	Layers
8 layers @ 0°	$[0_8]$	8
2 @ +45, 2 @ −45, symmetric	$[45_2/{-45_2}]_s$	8
+45/−45/+45/−45, symmetric	$[(\pm 45)_2]_s$	8
+45/−45/0/90, symmetric	$[\pm 45/0/90]_s$	8
50 groups of [±45/0/90], symmetric	$[(\pm 45/0/90)_{50}]_s$	400
+θ, −θ, symmetric	$[\pm\theta]_s$	4

Figure 22: Example laminates [16].

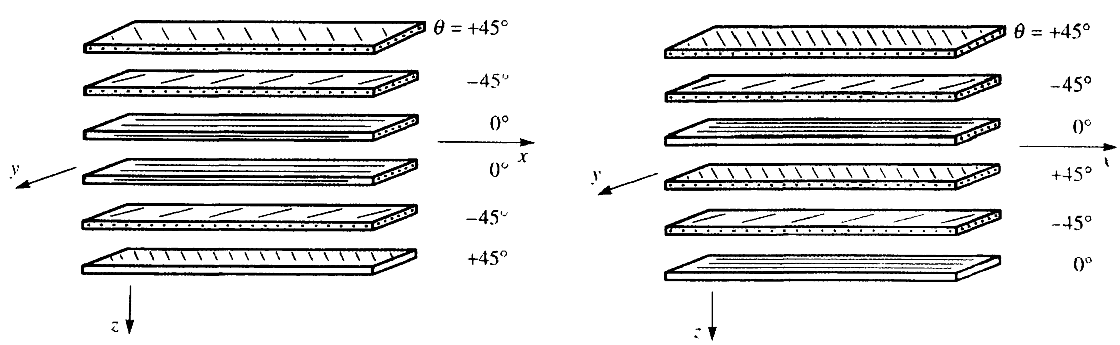

Figure 23: Example of a $[\pm 45/0]_S$ laminate [4].

An understanding of the sign convention, stacking sequence, and nomenclature of a fiber-reinforced laminate is necessary for the discussion of Classical Laminate Theory.

3.6 Classical Laminate Theory

The failure analysis of a fiber-reinforced laminate relies upon the development of the previously discussed topics. However, with the knowledge of these topics, Classical Laminate Theory can be developed and explored.

Classical Laminate Theory is used throughout the Aerospace Industry for the design of fiber-reinforced laminates. Knowledge of Classical Laminate Theory and its application is fundamental for the failure analysis of laminates.

3.6.1 The Kirchhoff Hypothesis

Classical Laminate Theory employs one of the principal hypotheses for the simplification of structural and material analysis [4]. The Kirchhoff Hypothesis, postulated in the mid 1800s by Gustav Robert Kirchhoff, theorizes the accurate response of beams, plates, and shells through calculations that can be applied to an array of other material types [4].

In a concise discussion, the Kirchhoff Hypothesis can be explained as a representation of lines that are straight and normal to the laminate's geometric midplane before laminate deformation [4]. A representation of this is given in Figure 24 as line AA'.

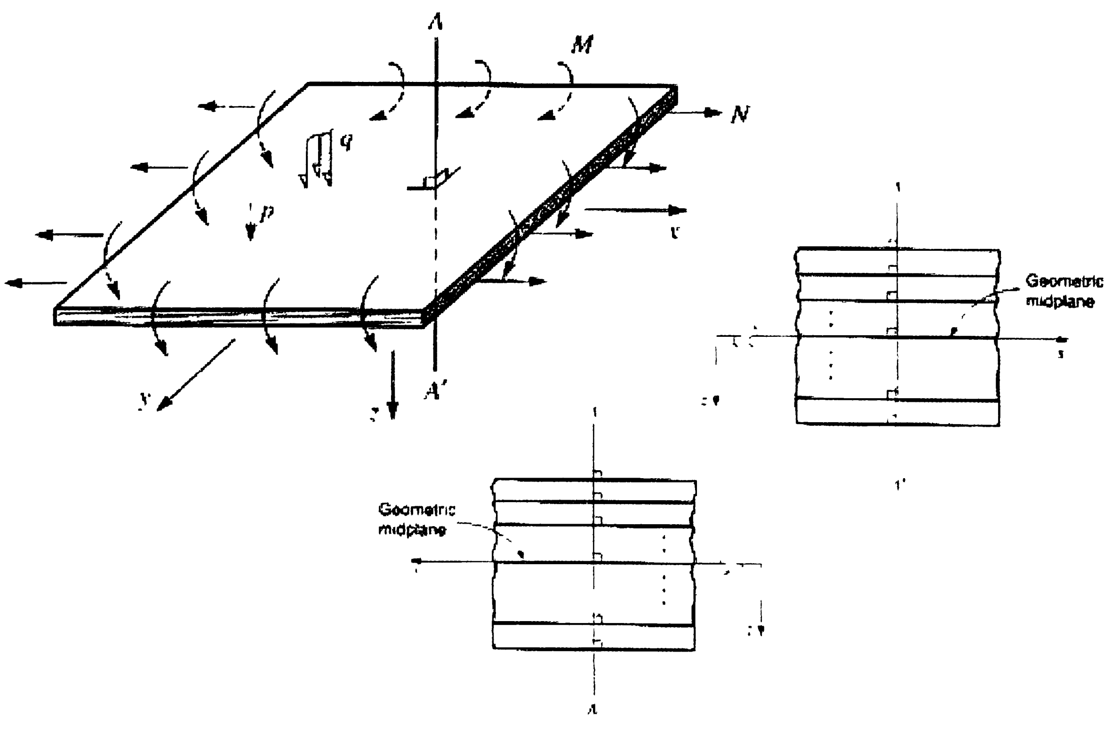

Figure 24: Illustration of undeformed normal AA' [4].

Furthermore, the Kirchhoff Hypothesis states that line AA' remains straight and normal to the geometric midplane regardless of laminate deformations, however, line AA' merely rotates and translates as a consequence [4]. A representation of the translation and rotation of line AA' is given in Figure 25.

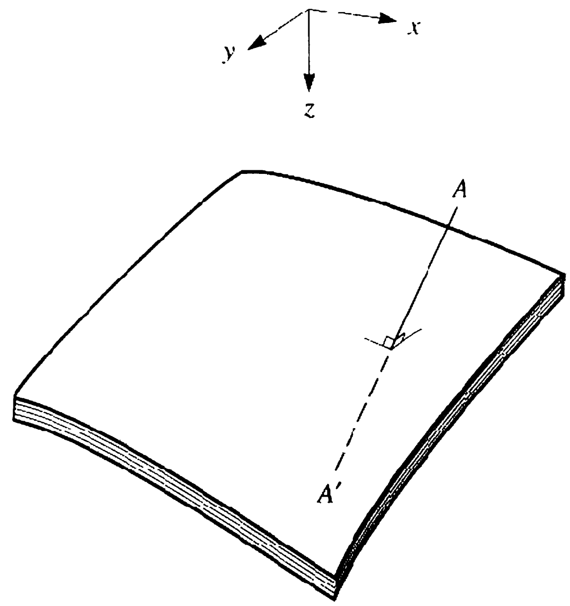

Figure 25: Illustration of deformed normal AA' [4].

Through the Kirchhoff Hypothesis, it can be derived that the displacements of an arbitrary point are given by [4]

$$\begin{aligned}
u(x,y,z) &= u^0(x,y) - z\frac{\partial w^0(x,y)}{\partial x} \\
v(x,y,z) &= v^0(x,y) - z\frac{\partial w^0(x,y)}{\partial y} \\
w(x,y,z) &= w^0(x,y)
\end{aligned} \tag{3.10}$$

It is important to note that the stress-strain relations of a laminate that are subsequently presented have been concluded based upon the application of the Kirchhoff Hypothesis. A detailed discussion of the Kirchhoff Hypothesis is available in *Reference 4.*

3.6.2 Laminate Strains

From the application of the Kirchhoff Hypothesis, the strain relationships of a laminate can be determined. The development of these laminate strain relationships is one of the most important assumptions of the Classical Laminate Theory [4].

Moreover, the laminate strain relations can be written as

$$\begin{aligned}
\varepsilon_x(x,y,z) &= \varepsilon_x^0(x,y) + z\kappa_x^0(x,y) \\
\varepsilon_y(x,y,z) &= \varepsilon_y^0(x,y) + z\kappa_y^0(x,y) \\
\gamma_{xy}(x,y,z) &= \gamma_{xy}^0(x,y) + z\kappa_{xy}^0(x,y)
\end{aligned} \tag{3.11}$$

where

$$\varepsilon_x^0(x,y)=\frac{\partial u^0(x,y)}{\partial x} \quad \text{and} \quad \kappa_x^0(x,y)=-\frac{\partial^2 w^0(x,y)}{\partial x^2}$$

$$\varepsilon_y^0(x,y)=\frac{\partial v^0(x,y)}{\partial y} \quad \text{and} \quad \kappa_y^0(x,y)=-\frac{\partial^2 w^0(x,y)}{\partial y^2} \tag{3.12}$$

$$\gamma_{xy}^0(x,y)=\frac{\partial v^0(x,y)}{\partial x}+\frac{\partial u^0(x,y)}{\partial y} \quad \text{and} \quad \kappa_{xy}^0(x,y)=-2\frac{\partial^2 w^0(x,y)}{\partial x \partial y}$$

3.6.3 Laminate Stresses

Now that the laminate strain relationships have been developed, the laminate stress-strain relations can be proposed. Using the strain relationships determined through the preceding application of the Kirchhoff Hypothesis, the previous stress-strain relationships for a laminate become [4]

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 + z\kappa_x^0 \\ \varepsilon_y^0 + z\kappa_y^0 \\ \gamma_{xy}^0 + z\kappa_{xy}^0 \end{Bmatrix} \tag{3.13}$$

3.6.4 Laminate Stress Distribution

With the development of the stress-strain relationships of a laminate now complete, a way of illustrating the layer-by-layer stresses in a laminate is needed. Therefore, the through thickness stress distribution of a laminate has become a valuable way of exemplifying these relations.

This type of illustrative tool can easily show the layer-by-layer stress differences within a laminate. Valuable insight can be obtained into whether or not individual ply materials and/or the laminate itself are a viable structural design for the applied loading conditions.

An example of a through thickness laminate stress distribution is given in Figure 26 and an example calculation has been provided in *Appendix B.3*.

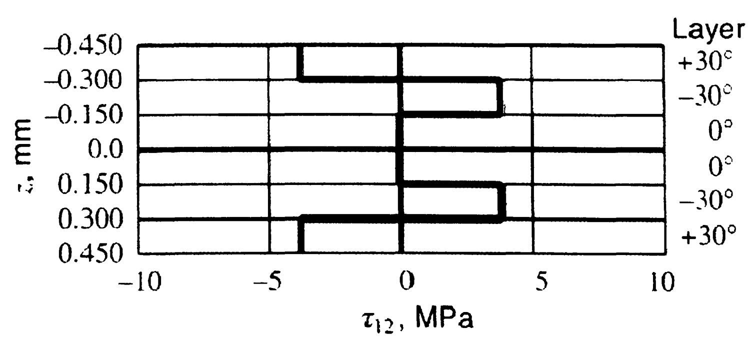

Figure 26: Example of through thickness stress distribution for σ_1 [4].

3.6.5 Force and Moment Resultants

Although the relations between stress and strain for a laminate have been addressed, the relationship between these and the applied forces and moments have yet to be introduced. These relationships are known as the force and moment resultants of the laminate.

The applied forces and moments required to produce the specified midplane deformations that were previously discussed are represented by integrals [4]. More specifically, the in-plane laminate force and moment resultants are given as

$$
\begin{aligned}
N_x &= \int_{-\frac{H}{2}}^{\frac{H}{2}} \sigma_x dz & \qquad M_x &= \int_{-\frac{H}{2}}^{\frac{H}{2}} \sigma_x z dz \\
N_y &= \int_{-\frac{H}{2}}^{\frac{H}{2}} \sigma_y dz & \qquad M_y &= \int_{-\frac{H}{2}}^{\frac{H}{2}} \sigma_y z dz \\
N_{xy} &= \int_{-\frac{H}{2}}^{\frac{H}{2}} \tau_{xy} dz & \qquad M_{xy} &= \int_{-\frac{H}{2}}^{\frac{H}{2}} \tau_{xy} z dz
\end{aligned}
\tag{3.14}
$$

where H is the laminate's total thickness. An illustration of a laminate subjected to these force and moment resultants can be seen in Figure 27 below.

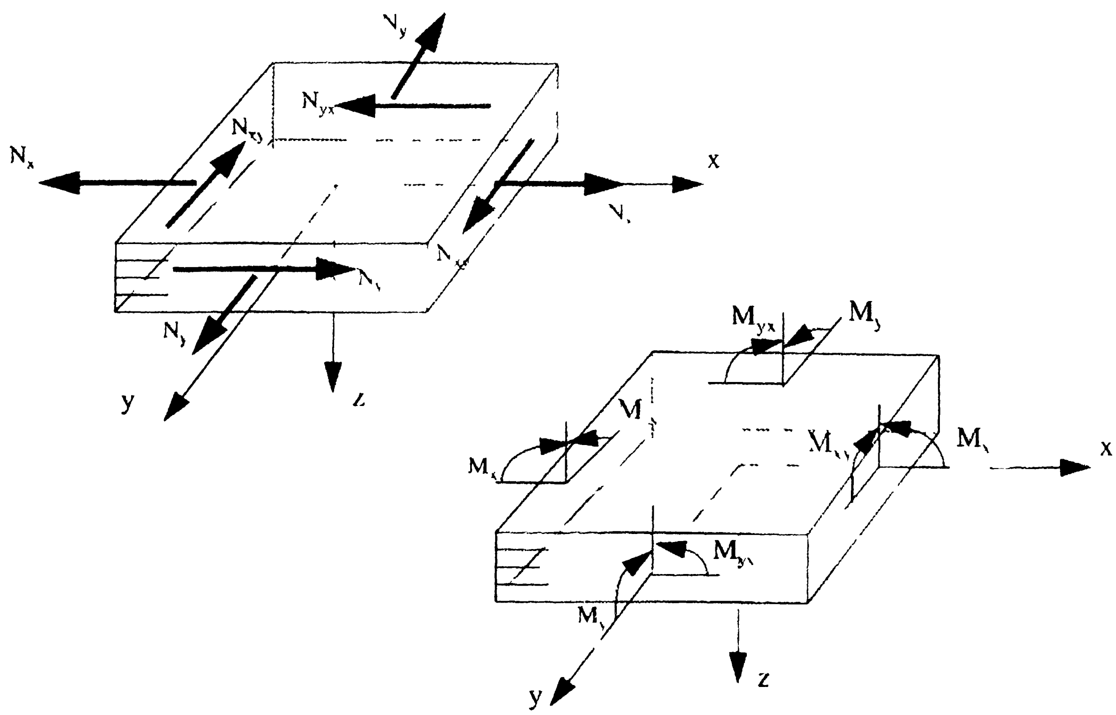

Figure 27: Laminate force and moment resultants [16].

3.6.6 Laminate Stiffness Matrix

Finally, the development of Classical Laminate Theory draws to an end with the introduction of the ABD matrix. Also known as the laminate stiffness matrix, the ABD matrix relates the laminate force and moment resultants to the laminate strains and curvatures [4]. This relationship, expressed in its formulaic form, is known as the constitutive equation of a laminate and can be written as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} \tag{3.15}$$

where

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{N} \bar{Q}_{ij,k}\left(z_k - z_{k-1}\right) \\ B_{ij} &= \frac{1}{2}\sum_{k=1}^{N} \bar{Q}_{ij,k}\left(z_k^2 - z_{k-1}^2\right) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{ij,k}\left(z_k^3 - z_{k-1}^3\right) \end{aligned} \tag{3.16}$$

It is important to note that the constitutive equation of a laminate given above illustrates how the force and moment resultants are coupled with the directional laminate strains and curvatures through the ABD matrix. These couplings are an important phenomenon of fiber-reinforced laminate design.

For an isotropic layer in a laminate, with thickness H, the ABD matrix values for the specified ply reduce to [4]

$$\begin{aligned} &A_{11} = A_{22} = \frac{EH}{1-\nu^2} = A \quad A_{12} = \nu\frac{EH}{1-\nu^2} = \nu A \quad A_{66} = \frac{EH}{2(1+\nu)} = \frac{1-\nu}{2}A \\ &D_{11} = D_{22} = \frac{EH^3}{12(1-\nu^2)} = D \quad D_{12} = \nu\frac{EH^3}{12(1-\nu^2)} = \nu D \quad D_{66} = \frac{EH^3}{24(1+\nu)} = \frac{1-\nu}{2}D \\ &D_{16} = D_{26} = 0 \quad A_{16} = A_{26} = 0 \end{aligned} \tag{3.17}$$

With the ABD matrix introduction being the concluding and most essential part of Classical Laminate Theory, the failure analysis of fiber-reinforced laminates can now be explored in the following chapter. A detailed understanding of the Classical Laminate

Theory concepts that have been introduced hitherto is fundamental for the subsequent discussion of fiber reinforced laminate failure analysis.

3.7 Effective Laminate Engineering Properties

Now that the stress-strain relations for a laminate can be expressed in terms of the global laminate coordinate system, it is useful to determine the effective engineering properties of the laminate. The effective engineering properties of a laminate can be determined through the use of the average laminate stresses.

The average laminate stresses, with thickness H, are defined as [4]

$$
\begin{aligned}
\bar{\sigma}_x &\equiv \frac{1}{H}\int_{-\frac{H}{2}}^{\frac{H}{2}} \sigma_x dz \\
\bar{\sigma}_y &\equiv \frac{1}{H}\int_{-\frac{H}{2}}^{\frac{H}{2}} \sigma_y dz \\
\bar{\tau}_{xy} &\equiv \frac{1}{H}\int_{-\frac{H}{2}}^{\frac{H}{2}} \tau_{xy} dz
\end{aligned}
\tag{3.18}
$$

which can also be expressed as

$$
\begin{aligned}
\bar{\sigma}_x &= \frac{1}{H}N_x \\
\bar{\sigma}_y &= \frac{1}{H}N_y \\
\bar{\tau}_{xy} &= \frac{1}{H}N_{xy}
\end{aligned}
\tag{3.19}
$$

Furthermore, using these average laminate stresses, the effective laminate engineering properties can be defined as [4]

$$
\begin{aligned}
\bar{E}_x &\equiv \frac{A_{11}A_{22} - A_{12}^2}{A_{22}H} \\
\bar{E}_y &\equiv \frac{A_{11}A_{22} - A_{12}^2}{A_{11}H} \\
\bar{G}_{xy} &\equiv \frac{A_{66}}{H} \\
\bar{\nu}_{xy} &\equiv \frac{A_{12}}{A_{22}} \\
\bar{\nu}_{yx} &\equiv \frac{A_{12}}{A_{11}}
\end{aligned}
\tag{3.20}
$$

3.8 Laminate Classification

Since the ABD matrix is not trivial, special cases in which the specific orientation of a laminate can further reduce the ABD matrix have been developed. These special cases are known as laminate classifications. There are five common classifications of a laminate. These classifications pertain to their layer orientation and are as follows:

1. *Symmetric Laminates*: A laminate is classified as symmetric if every layer, on one side of the laminate's midplane, with specific material properties, thickness, and fiber orientation, corresponds to another opposing layer on the other side of the laminate's midplane with the same material properties, thickness, and fiber orientation [4]. For example, a [0/90/90/0] laminate is symmetric and can be written as $[0/90]_S$. For the case of a symmetric laminate, the ABD matrix reduces so that all components of the B matrix equal zero [4].

2. *Balanced Laminates*: A laminate is classified as balanced if every layer with specific material properties, thickness, and fiber orientation, corresponds to another layer with the same material properties and thickness but opposite fiber orientation within the laminate [4]. For example, a [±45/±30] laminate is considered to be balanced. For the case of a balanced laminate, the ABD matrix reduces so that the A_{16} and A_{26} components of the A matrix equal zero [4].

3. *Symmetric Balanced Laminates*: A laminate is classified as symmetric and balanced if it qualifies as being both symmetric and balanced per the previously established definitions. For example, a $[\pm45/\pm30]_S$ laminate is considered to be a symmetric balanced laminate. For the case of a symmetric balanced laminate, the ABD matrix reduces so that all components of the B matrix as well as the A_{16} and A_{26} components of the A matrix are equal to zero [4].

4. *Cross-Ply Laminates*: A laminate is classified as cross-ply if every layer within the laminate has a fiber orientation of 0° or 90° [4]. For example, a [±90/0] laminate is considered to be cross-ply. For the case of a cross-ply laminate, the ABD matrix reduces so that the A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, and D_{26} components are equal to zero [4].

5. *Symmetric Cross-Ply Laminates*: A laminate is classified as symmetric and cross-ply if it qualifies as being both symmetric and cross-ply per the previously established definitions. For example, a $[\pm90/0]_S$ laminate is considered to be a symmetric cross-ply laminate. For the case of a symmetric cross-ply laminate, the ABD matrix reduces so that all components of the B matrix as well as the A_{16}, A_{26}, D_{16}, and D_{26} components are equal to zero [4].

CHAPTER 4

Laminate Failure Theories and Mechanisms

If fiber-reinforced composite materials are to be used in structural applications, a set of criteria that can be used as a guideline for mechanical failure must be established. Although there are dozens of failure theories that have been established to address this issue, there are only two that are extensively used through the Aerospace Industry for failure analysis of fiber-reinforced laminates; the Maximum Stress and Tsai-Wu criteria.

Although it is helpful to model the failure stress of a laminate, there are so many factors that determine a laminate's viability. The manufacturing and handling processes of a laminate structure can appear to be equivalent for each specimen but, through macroscopic and microscopic inspection, inconsistencies between specimens are always prevalent. For this reason, it is important to not only model a laminate idealistically through failure theory criterion models but also to perform extensive laboratory testing of manufactured laminate specimens.

Nonetheless, the use of failure criterion models is helpful for the design and analysis of fiber-reinforced laminates before any manufacturing has begun. This can save a lot of time and money in the development of fiber-reinforced structural laminates.

Furthermore, when designing fiber-reinforced laminates, it is imperative to understand that failure theories are usually application specific and therefore should be used appropriately to avoid inaccurate analyses.

4.1 Introduction to Laminate Failure Theories and Mechanisms

When it comes to developing failure criterion models for fiber-reinforced laminates, one must realize that the model being presented must be developed based on the anisotropy of a composite laminate. Further simplification can then be made for application specific criterion models. Each failure theory presents a set of conditions that stipulate the onset of failure. These conditions are known as a failure criterion.

The subsequent sections in this chapter present several failure criteria that were used in this research in order to provide decent selection for use in the CFA software being developed (discussed in the next chapter). However, it is important to remember that the Tsai-Wu and Maximum Stress failure criteria are used predominantly throughout the Aerospace Industry for fiber-reinforced laminate failure analysis.

There are two characteristics of failure criterion models that should be discussed: 1) The ability to predict the onset of failure, and 2) The ability to identify the corresponding mode of failure. For example, the Tsai-Wu Failure Criterion offers only a way of satisfying the former and no way of satisfying the later. On the other hand, the Hashin Failure Criterion has the ability to satisfy both.

When designing a fiber-reinforced laminate, it can be beneficial to use multiple failure criteria in order to try and obtain a conservative failure model. However, since there is a multitude of failure criteria that have been developed, one must choose criteria that are specific to the application of the laminate in question.

It is also important to remember that failure analysis should not be used in the absence of specimen testing; rather, they should coexist throughout the development of a fiber-reinforced laminate structure.

Furthermore, for the purposes of this research, all failure criteria subsequently presented are given with the assumption of a state of plane-stress. Moreover, the following failure criteria are presented so that failure will occur if the inequalities are not satisfied.

4.2 Laminate Failure Theories

The following sections present multiple failure theories that are used in the development of the CFA software to be presented in the next chapter. Moreover, these failure criteria are used throughout the Aerospace Industry for the design of fiber-reinforced laminates. Comprehension of these failure theories is essential for the failure analysis of laminates.

4.2.1 Maximum Stress Failure Criterion

The Maximum Stress Failure Criterion is predominantly used in the analysis and design of isotropic materials. However, it is also used often in the analysis and design of fiber-reinforced laminates due to its strong relation to the material's mechanical properties. As its name implies, this criterion is derived from the material's maximum allowable stresses before failure occurs.

For a laminate, the Maximum Stress Criterion is used to relate each layer's maximum allowable stresses to the stresses calculated in each layer for the applied loading set. Therefore, this criterion is represented as [4]

$$\begin{aligned} \sigma_1 &< \sigma_1^T, \sigma_1^C \\ \sigma_2 &< \sigma_2^T, \sigma_2^C \\ \tau_{12} &< \tau_{12}^F, \tau_{12}^{-F} \end{aligned} \tag{4.1}$$

where the T, C, and F notations stand for tension, compression, and failure, respectively.

4.2.2 Maximum Strain Failure Criterion

The Maximum Strain Failure Criterion is occasionally used in the analysis and design of isotropic materials. Although this criterion has strong relations to the material's mechanical properties, as does the Maximum Stress Criterion, its use in the analysis and design of fiber-reinforced laminates is less frequent in comparison. As its name implies, this criterion is derived from the material's maximum allowable strains before failure.

For a laminate, the Maximum Strain Criterion is used to relate each layer's maximum allowable strains to the strains calculated in each layer for the applied loading set. Therefore, this criterion conceptually similar to the Maximum Stress Failure Criterion and can be derived as [4]

$$\begin{aligned} \varepsilon_1 &= \frac{\sigma_1}{E_1} - \frac{\nu_{12}\sigma_2}{E_1} < \varepsilon_1^T, \varepsilon_1^C \\ \varepsilon_1 &= \frac{\sigma_2}{E_2} - \frac{\nu_{12}\sigma_1}{E_2} < \varepsilon_2^T, \varepsilon_2^C \\ \gamma_{12} &= \frac{\tau_{12}}{G_{12}} < \gamma_{12}^F, \gamma_{12}^{-F} \end{aligned} \tag{4.2}$$

4.2.3 Extended von Mises Failure Criterion

Although the von Mises Failure Criterion is extensively used for the analysis and design of isotropic materials, it has subsequently been adapted for use with anisotropic materials including laminates. There are many different adaptations of the original von Mises Failure Criterion for isotropic materials as will be presented in the following sections.

The original von Mises Failure Criterion for isotropic materials has been modified for use with fiber-reinforced laminates. This extended von Mises Failure Criterion is representative of a conservative failure criterion as will be illustrated and discussed in the following chapter. With the von Mises failure criterion given as [3]

$$(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + 6\left(\tau_{23}^2 + \tau_{31}^2 + \tau_{12}^2\right) = 2\sigma_{yp}^2 \tag{4.3}$$

where the notation *yp* stands for yield point, the Extended von Mises Failure Criterion can be developed through

$$
\begin{aligned}
&(\sigma_2-0)^2+(0-\sigma_1)^2+(\sigma_1-\sigma_2)^2+6\left(0+0+\tau_{12}^2\right)=2\sigma_{yp}^2\\
&(\sigma_1-\sigma_2)^2+\sigma_2{}^2+\sigma_1{}^2+6\tau_{12}^2=2\sigma_{yp}^2\\
&\frac{(\sigma_1-\sigma_2)^2}{2\sigma_1^y\sigma_2^y}+\frac{\sigma_2{}^2}{2\left(\sigma_2^y\right)^2}+\frac{\sigma_1{}^2}{2\left(\sigma_1^y\right)^2}+\frac{3\tau_{12}^2}{\left(\tau_{12}^y\right)^2}=1
\end{aligned}
\tag{4.4}
$$

where the notation y stands for yield.

Finally, the Extended von Mises Failure Criterion can be written as

$$
\begin{aligned}
&\frac{(\sigma_1-\sigma_2)^2}{2\sigma_1^T\sigma_2^T}+\frac{\sigma_2{}^2}{2\left(\sigma_2^T\right)^2}+\frac{\sigma_1{}^2}{2\left(\sigma_1^T\right)^2}+\frac{3\tau_{12}^2}{\left(\tau_{12}^F\right)^2}<1\\
&\frac{(\sigma_1-\sigma_2)^2}{2\sigma_1^C\sigma_2^C}+\frac{\sigma_2{}^2}{2\left(\sigma_2^C\right)^2}+\frac{\sigma_1{}^2}{2\left(\sigma_1^C\right)^2}+\frac{3\tau_{12}^2}{\left(\tau_{12}^{-F}\right)^2}<1
\end{aligned}
\tag{4.5}
$$

4.2.4 Hashin Failure Criterion

The Hashin Failure Criterion is application specific, meaning it was developed for use specifically with fiber-reinforced composites. This criterion provides a clear-cut way for an engineer to analyze and design fiber-reinforced laminates with a key benefit of the Hashin Failure Criterion being that it can inherently indicate a laminate's mode of failure.

This criterion is represented through five equations that each relate to a different mode of mechanical failure. These relations are given as [10]

$$
\begin{aligned}
&F^T=\left(\frac{\sigma_1}{\sigma_1^T}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^F}\right)^2<1\\
&F^C=\left(\frac{\sigma_1}{\sigma_1^C}\right)^2<1\\
&M^T=\left(\frac{\sigma_2}{\sigma_2^T}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^F}\right)^2<1\\
&M^C=\left(\frac{\sigma_2}{\sigma_2^C}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^{-F}}\right)^2<1\\
&FMS^C=\left(\frac{\sigma_1}{\sigma_1^C}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^{-F}}\right)^2<1
\end{aligned}
\tag{4.6}
$$

where F^T, F^C, M^T, M^C, and FMS^C notations stand for fiber tensile, fiber compressive, matrix tensile, matrix compressive, and fiber-matrix shear failures, respectively.

4.2.5 Hill Failure Criterion

The Hill Failure Criterion is another criterion developed for the analysis of anisotropic fiber-reinforced laminates. As another interpretation of the von Mises Failure Criterion, the Hill Failure Criterion is represented in a tensile and compressive analysis form.

This criterion is given as [11]

$$F\sigma_2^2 + G\sigma_1^2 + H(\sigma_1 - \sigma_2)^2 + 2N\tau_{12}^2 < 1 \tag{4.7}$$

where

$$\begin{aligned} F &= \frac{1}{2}\left[\frac{1}{(\sigma_2^y)^2} - \frac{1}{(\sigma_1^y)^2}\right] \\ G &= \frac{1}{2}\left[\frac{1}{(\sigma_1^y)^2} - \frac{1}{(\sigma_2^y)^2}\right] \\ H &= \frac{1}{2}\left[\frac{1}{(\sigma_1^y)^2} + \frac{1}{(\sigma_2^y)^2}\right] \\ N &= \frac{1}{2(\tau_{12}^y)^2} \end{aligned} \tag{4.8}$$

4.2.6 Tsai-Hill Failure Criterion

The Tsai-Hill Failure Criterion is an extension of the Hill Failure Criterion. Assuming that the layers of a laminate are transversely isotropic, the Hill Failure Criterion can be simplified and is known as the Tsai-Hill Failure Criterion.

First proposed by Azzi and Tsai in 1965, this criterion is given as [12]

$$\left(\frac{\sigma_1}{\sigma_1^y}\right)^2 + \left(\frac{\sigma_2}{\sigma_2^y}\right)^2 - \frac{\sigma_1\sigma_2}{(\sigma_1^y)^2} + \left(\frac{\tau_{12}}{\tau_{12}^F}\right)^2 < 1 \tag{4.9}$$

4.2.7 Tsai-Wu Failure Criterion

The Tsai-Wu Failure Criterion is perhaps the most widely used failure criterion for the analysis and design of fiber-reinforced laminates. Although this criterion is not trivial in its representation, it seems to be extensively used throughout the Aerospace Industry.

Postulated by Tsai and Wu in 1971, this criterion is given as [5]

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 - \sqrt{F_{11}F_{22}}\sigma_1\sigma_2 < 1 \tag{4.10}$$

where

$$\begin{aligned} F_1 &= \frac{1}{\sigma_1^T} + \frac{1}{\sigma_1^C} \qquad F_2 = \frac{1}{\sigma_2^T} + \frac{1}{\sigma_2^C} \qquad F_{11} = \frac{1}{\sigma_1^T \sigma_1^C} \\ F_{22} &= \frac{1}{\sigma_2^T \sigma_2^C} \qquad F_{66} = \frac{1}{\left(\tau_{12}^F\right)^2} \end{aligned} \tag{4.11}$$

4.2.8 Hoffman Failure Criterion

The Hoffman Failure Criterion is another extension of the Hill Failure Criterion to incorporate the ability to model different strengths in tension and compression. As with the Tsai-Wu Failure Criterion, the Hoffman Failure Criterion has the ability to calculate the onset of failure but is incapable of determining the mode of failure.

Formulated in 1967 by Hoffman, this criterion is given as [13]

$$C_1\sigma_1^2 + C_2\sigma_2^2 - C_3\sigma_1\sigma_2 + C_4\sigma_1 + C_5\sigma_2 + C_9\tau_{12}^2 < 1 \tag{4.12}$$

where

$$\begin{aligned} C_1 &= \frac{1}{\sigma_1^T \sigma_1^C} \\ C_2 &= \frac{1}{\sigma_2^T \sigma_2^C} \\ C_3 &= -\left(\frac{1}{\sigma_1^T \sigma_1^C} + \frac{1}{\sigma_2^T \sigma_2^C}\right) \\ C_4 &= \frac{1}{\sigma_1^T} - \frac{1}{\sigma_1^C} \\ C_5 &= \frac{1}{\sigma_2^T} - \frac{1}{\sigma_2^C} \\ C_9 &= \frac{1}{\left(\tau_{12}^F\right)^2} \end{aligned} \tag{4.13}$$

4.3 Laminate Failure Mechanisms

When a laminate is subjected to loading conditions that initiate failure, the laminate can fail in a variety of different ways. These mechanisms of failure are referred to as failure modes. Fiber-reinforced laminates, due to their fibrous composition, have unique failure modes that are not applicable to isotropic materials. Since a fiber-reinforced laminate is an anisotropic material, failure can be inconsistent and incorporate few or multiple modes of failure. For the purposes of this research it is useful to present five common modes of failure for fiber-reinforced composites and laminates.

1. *Fiber-Matrix Interface Debonding*: This type of failure mechanism occurs when the matrix starts to debond at its fiber interface. A large stress concentration can cause the fiber to break if debonding continues. An example of the Fiber-Matrix Interface Debonding failure mode is given in Figure 28.

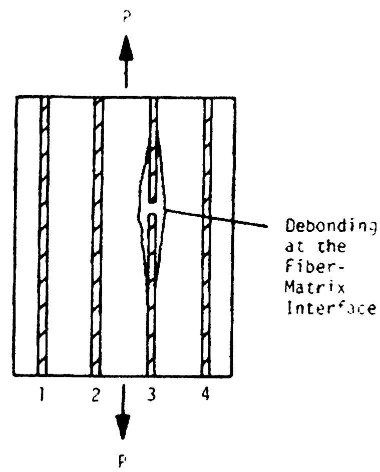

Figure 28: Illustration of fiber-matrix interface debonding [14].

2. *Matrix Cracking*: This type of failure mechanism occurs when the matrix within a composite develops fractures. These fractures can then propagate into larger cracks. If this crack meets a fiber-matrix interface, a large stress concentration can develop causing the fiber to break. An example of the Matrix Cracking failure mode is given in Figure 29.

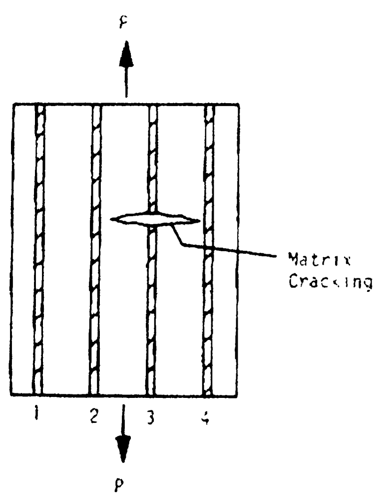

Figure 29: Illustration of matrix cracking [14].

3. *Fiber Breakage*: This type of failure mechanism occurs when the ultimate tensile strength of the fibers is exceeded. However, matrix selection can be a tricky game. If the matrix is too strong then it will not deform and it can cause high stress concentrations. However, a strong matrix can overcome fiber breaks because it has the ability to transfer the loads between broken fibers. Conversely, if the matrix is too ductile then it will deform too much and not be able to transfer the loads between broken fibers. However, a ductile matrix can eliminate the potential for matrix cracking. An example of the Fiber Breakage failure mode is given in Figure 30.

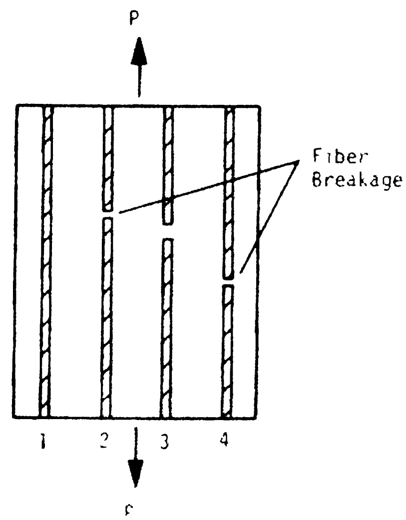

Figure 30: Illustration of fiber breakage [14].

4. *Fiber Pullout*: This type of failure mechanism occurs after fiber breakage has occurred. If the matrix develops a crack which opens under stress, the broken fiber can be pulled out if there is an insufficient fiber-matrix interface bond.

5. *Delamination*: This type of failure mechanism occurs when the layers within the laminate are not bonded sufficiently to sustain the interlaminar stresses. This type of failure mode can be devastating and can be avoided by using a sufficient bonding method between layers. Voids within the bond and inconsistencies in manufacturing are often to blame for this mode of failure.

Moreover, it is typical that multiple failure modes occur at the same time in a composite. A composite laminate can withstand many different failure mode occurrences and still not fail entirely. This is an inherent attribute that makes the use of composites so desirable. However, the fiber-matrix design often makes it difficult to visually discern any regions of failure within a laminate. Inspection usually relies upon non-destructive testing techniques to determine if a laminate structure is intact.

CHAPTER 5

CFA – An Intelligible Software Approach

Although useful formulaic methods have been developed for analyzing fiber-reinforced laminates, these calculations can be quite tedious when they are used in an iterative design process. Therefore, development of software that can conduct computer-aided laminate failure analysis can provide an indispensable tool for the design of composites in the Aerospace Industry.

Engineers rely more and more on the use of technology to design materials and structures that would otherwise require additional time to accomplish. With the exponential growth of technology in the past few decades, the demand for intuitive engineering software is ever increasing. Used throughout the Aerospace Industry, software such as CATIA®, MatLab®, NEiNastran®, ANSYS®, and many others have been developed to meet this demand.

With the younger generations of engineers being more and more receptive and adaptive to these advances in technology, it is apparent that it is important to familiarize engineers with these types of software near the beginning in their educational careers. The ability for engineers to use the technology that is available to them is invaluable as it can ultimately boost their efficiency.

This research develops educational software that provides a means of performing fiber-reinforced laminate failure analysis. CFA, as it has been branded, will be introduced in the subsequent sections of this chapter.

5.1 Introduction to CFA

It is the goal of this research to develop an intelligible way to provide quick and reliable composite laminate failure analysis using multiple failure criteria and to illustrate a piecewise representation of the Tsai-Wu Failure Criterion. Without a doubt, it is obvious that an in depth software package is the solution. Hence, a composite failure analysis program, CFA, is introduced.

Development of CFA will provide a platform for the advancement of composite materials education at Embry-Riddle Aeronautical University. Therefore, it is the objective of this research to provide a comprehensive software package to further composite materials education at the university.

The development, validation, and capabilities of CFA will be subsequently discussed. However, it is important to note that this research not only develops the CFA software package but it also introduces a technique for the representation of failure envelopes. This is given by a piecewise representation of the Tsai-Wu Criterion which ultimately can allow the designer to effortlessly interpret a failure envelope of a composite laminate.

Further discussion about CFA and the piecewise representation of the Tsai-Wu Criterion developed in this research is presented next.

5.2 Software Limitations and Assumptions

The analysis performed through this software is governed by a few limitations and assumptions that were previously discussed in *Section 1.3*. Furthermore, this software was developed in Excel® and MatLab® and requires a working installation of these programs to function. Therefore, it is crucial for the user to be proficient in these programs in order to properly perform fiber-reinforced laminate failure analysis using CFA. The user is responsible for being familiar with these assumptions and limitations.

Moreover, CFA is limited to its current application and is intended to provide failure analysis of fiber-reinforced laminates for educational use. By no means is CFA intended to provide results that lead to the design, manufacture, or implementation into an industrial application.

5.3 CFA Development and Overview

CFA was developed in order to provide computer aided design and failure analysis of fiber-reinforced laminates. Originally, CFA was to be packaged as a single program written in the Java computing language. After five months of developing CFA in Java, it became evident that the program would take a prolonged period of time to complete. Consequently, a decision was made to abandon the five months of CFA development in Java and start from scratch through an intelligible integration of Excel® and MatLab®.

Using Excel® and MatLab® as a base for the development of the CFA software package, a concise and lucid application of computer aided design and failure analysis of fiber-reinforced laminates was developed. Excel® was used to conduct fiber-reinforced laminate design, stress distribution, and failure analysis. MatLab® was used to provide graphical failure envelopes for the failure analysis obtained from Excel®. Since the Excel® and MatLab® programs are widely used throughout most engineering disciplines; they are an excellent tool for the development of CFA.

Please refer to the analysis tutorial, graphing tutorial, and user manual for proper instruction on the use of CFA. Links to these items can be found on the first sheet of the CFA Excel® workbook labeled "CFA" as seen in Figure 31.

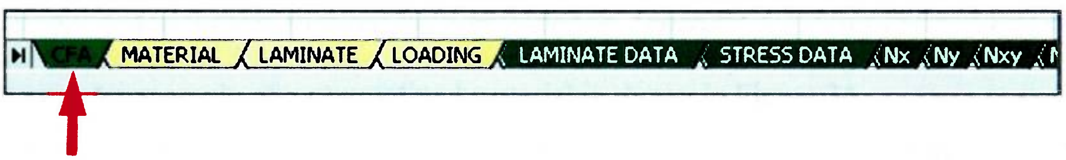

Figure 31: "CFA" sheet in CFA Excel® workbook.

The links are located in the upper left portion of the Excel® sheet as shown in Figure 32.

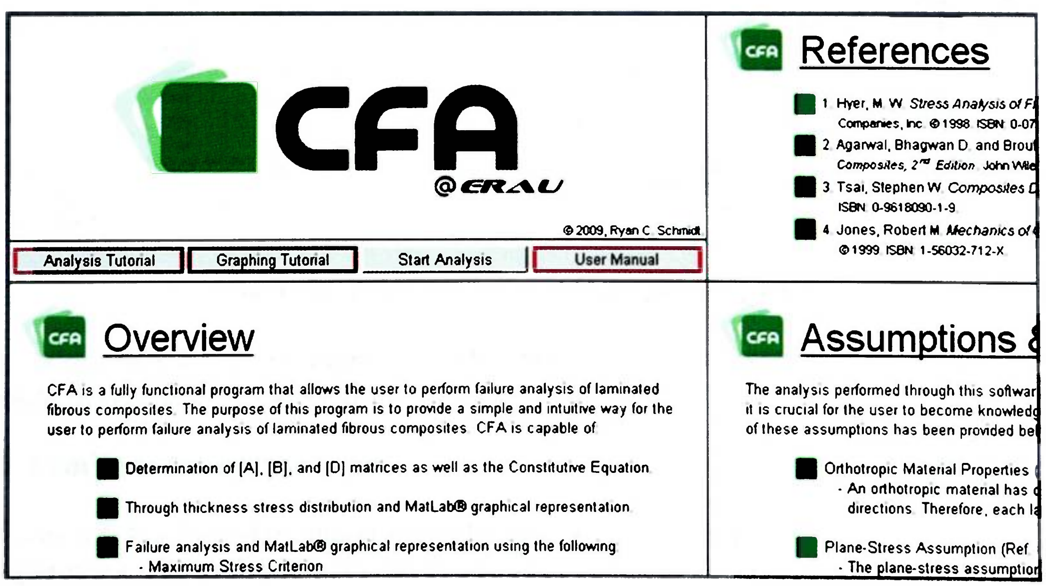

Figure 32: CFA instructional links.

An overview of CFA's development, validation, and capabilities is now presented. Illustrations such as those given above are taken from CFA and incorporated in the following sections for explicatory purposes.

5.3.1 Material Properties Input

When designing a composite laminate, it is necessary to define the materials that will be used for the laminate's layers. Therefore, the second sheet of the CFA Excel® workbook labeled "MATERIAL", seen in Figure 33, gives some preloaded examples of materials and also allows the user to define up to ten different custom materials for use within the laminate.

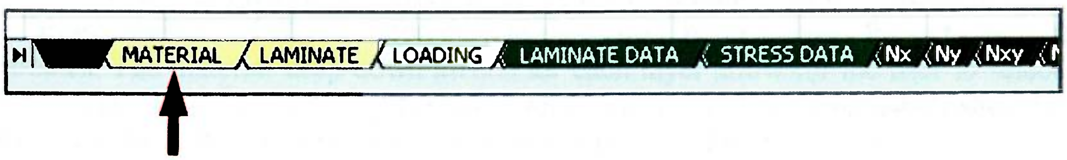

Figure 33: "MATERIAL" sheet of CFA Excel® workbook.

Material properties such as elastic constants, material strengths, and thickness are important definitions that are used throughout the CFA failure analysis software. A table of sample material properties has been provided in *Appendix A.1*. User defined materials allow for the use of different or new materials that are not previously defined within the software. An example of a user defined material is shown in Figure 34.

					Elastic Constants				Strengths				
Fiber / Matrix	Type	Thickness, Δh [mm]	FVF, V_f	Density, ρ [g/cm³]	E_1 [GPa]	E_2 [GPa]	ν_{12}	G_{12} [GPa]	σ_1 [MPa]	σ'_1 [MPa]	σ_2 [MPa]	σ'_2 [MPa]	τ_{12} [MPa]
T300 / N5208	CFRP	0.125	0.700	1.600	181.00	10.30	0.280	7.17	1500.00	-1500.00	40.00	-246.00	68.00
AS / 3501	CFRP	0.125	0.660	1.600	138.00	8.96	0.300	7.10	1447.00	-1447.00	51.70	-206.00	93.00
AS4 / PEEK APC2	CFRTP	0.125	0.660	1.600	134.00	8.90	0.280	5.10	2130.00	-1100.00	80.00	-200.00	160.00
H-IM6 / Epoxy	CFRP	0.125	0.660	1.600	203.00	11.20	0.320	8.40	3500.00	-1540.00	56.00	-150.00	98.00
T300 / F934 (4mil tape)	CFRP	0.100	0.600	1.500	148.00	9.65	0.300	4.55	1314.00	-1220.00	43.00	-168.00	48.00
B(4) / N5505	BFRP	0.125	0.500	2.000	204.00	18.50	0.230	5.59	1260.00	-2500.00	61.00	-202.00	67.00
E-Glass / Epoxy	GFRP	0.125	0.450	1.800	38.60	8.27	0.260	4.14	1062.00	-610.00	31.00	-118.00	72.00
Kevlar 49 / Epoxy	KFRP	0.125	0.600	1.460	76.00	5.50	0.340	2.30	1400.00	-235.00	12.00	-53.00	34.00
T300 / F934 (13mil cloth)	CCRP	0.325	0.600	1.500	74.00	74.00	0.050	4.55	499.00	-352.00	458.00	-352.00	46.00
T300 / F934 (7mil cloth)	CCRP	0.175	0.600	1.500	66.00	66.00	0.040	4.10	375.00	-279.00	368.00	-278.00	46.00
Example Material	CFRP	0.150			155.00	12.10	0.248	4.40	1500.00	-1250.00	50.00	-200.00	100.00
Custom Fiber / Matrix 2													
Custom Fiber / Matrix 3													

Figure 34: Example of user defined material in CFA.

This feature of CFA gives important information about the materials to be used for the laminate design in the next section.

5.3.2 Laminate Properties Input

Now that CFA's material database has been discussed, a way to design a laminate with multiple plies and multiple material types must be created. Therefore, the third sheet of the CFA Excel® workbook labeled "LAMINATE", shown in Figure 35, allows the user to design a custom fiber-reinforced laminate.

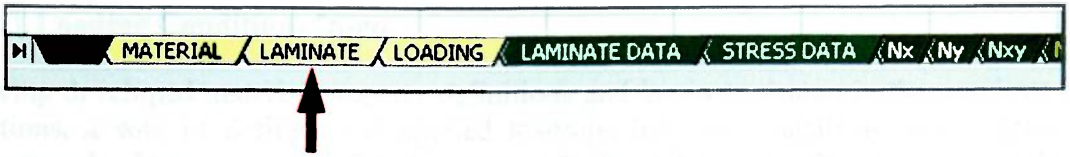

Figure 35: "LAMINATE" sheet in CFA Excel® workbook.

Laminate properties such as the number of layers, layer fiber orientation, and the layer material are selected by the user for a specific laminate design. For convenience, CFA allows for the number of layers and their respective materials to be chosen from drop-down menus. CFA is capable of analyzing a laminate comprised of up to 500 layers. However, it is important to note that some of the assumptions and or methods used for the failure analysis of fiber-reinforced laminates may be erroneous for extremely thick laminates. The material drop-down menus for each layer allow for the user to select any of the predefined or custom fiber-reinforced materials that were established in the previous section. An example laminate design is given in Figure 36.

Select Number of Layers: 10

Layer	Angle [°]	Thickness [m]	Material	z values	
				z0	-0.00075
1	20	0.00015	Example Material	z1	-0.0006
2	-20	0.00015	Example Material	z2	-0.00045
3	0	0.00015	Example Material	z3	-0.0003
4	0	0.00015	Example Material	z4	-0.00015
5	0	0.00015	Example Material	z5	-5.4E-20
6	0	0.00015	Example Material	z6	0.00015
7	0	0.00015	Example Material	z7	0.0003
8	0	0.00015	Example Material	z8	0.00045
9	-20	0.00015	Example Material	z9	0.0006
10	20	0.00015	Example Material	z10	0.00075

Figure 36: Demonstration of laminate design in CFA.

This feature of CFA allows for the user to easily create a laminate for fiber-reinforced laminate failure analysis providing the user with the ability to perform an iterative laminate development.

5.3.3 Loading Conditions Input

Having developed material property definitions and laminate design in the previous two sections, a way of defining the applied loadings for failure analysis testing must be introduced. Therefore, the fourth sheet of the CFA Excel® workbook labeled "LOADING", shown in Figure 37, allows the user to specify the laminate loading conditions.

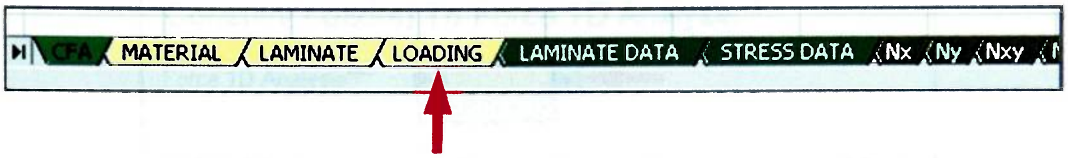

Figure 37: "LOADING" sheet in CFA Excel® workbook.

Firstly, it is necessary to decide which type of failure analysis is going to be performed. In other words, are you analyzing a tube or a laminate? CFA allows the user to specify the analysis type through a drop-down selection illustrated in Figure 38.

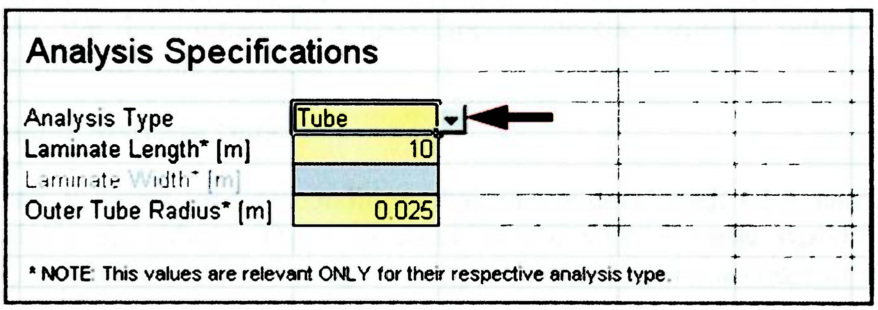

Figure 38: Choosing the CFA analysis type.

Secondly, CFA relies on the user to input force and moment resultants that correspond to a set of specific loading conditions. It is a requirement of the user to be able to translate applied forces and moments into their equivalent force and moment resultants that are to be used for failure analysis. For example, these force and moment resultants are entered as shown in Figure 39.

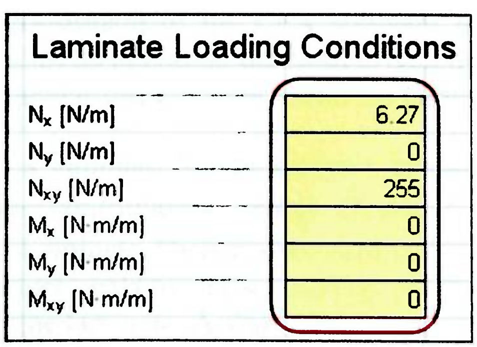

Figure 39: Entering force and moment resultants.

Lastly, CFA provides the ability to force a 1D failure analysis. This means that the user can determine what loads are required for failure while incorporating existing applied loads that are not the subject of investigation. For example, if a constant load was applied in the y-direction, what would be the failure load in the x-direction? Isolation of specific loads can be very useful when designing a laminate. This option is controlled in CFA by a drop-down menu, seen in Figure 40, which denotes a force 1D failure analysis Boolean.

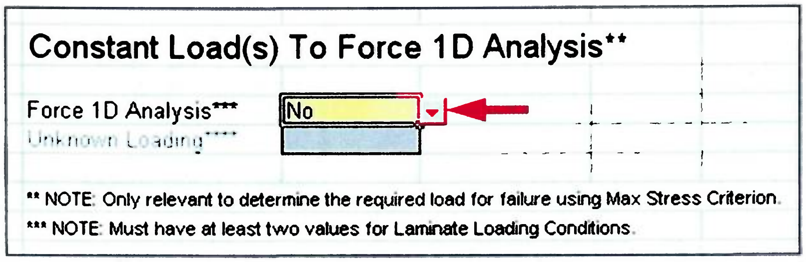

Figure 40: Forcing 1D failure analysis in CFA.

Furthermore, in order to obtained meaningful failure analysis results, it is important for the user to comply with the units specified throughout the CFA Excel® workbook. With this in mind, the development of CFA's fiber-reinforced laminate failure analysis is presented in the following sections.

5.3.4 Laminate Matrices Data

Now that the laminate loading conditions have been developed, CFA can execute its failure analysis algorithms. The fifth sheet of the CFA Excel® workbook labeled "LAMINATE DATA", shown in Figure 41, provides a detailed account of all laminate matrices data calculated during the initial phase of CFA's laminate failure analysis.

Figure 41: "LAMINATE DATA" sheet in CFA Excel® workbook.

Perhaps the most useful constituent of the CFA software is the data that is obtained throughout its execution. With its operating platform driven by Excel®, CFA computes a plethora of data which, in tabular form, is very useful for laminate failure analysis.

The initial phase of CFA's laminate failure analysis includes the calculation of each layer's individual ABD layer contributions, and Q, $\bar{Q}$, and T matrices data. Through the combined use of Classical Laminate Theory, presented in *Section 3.6*, and the foregoing discussions of laminate failure analysis, these matrices are independently calculated and displayed for each layer of the laminate. A single layer representation of the matrices data obtained through these calculations has been extrapolated and provided as an illustrative example shown in Figure 42.

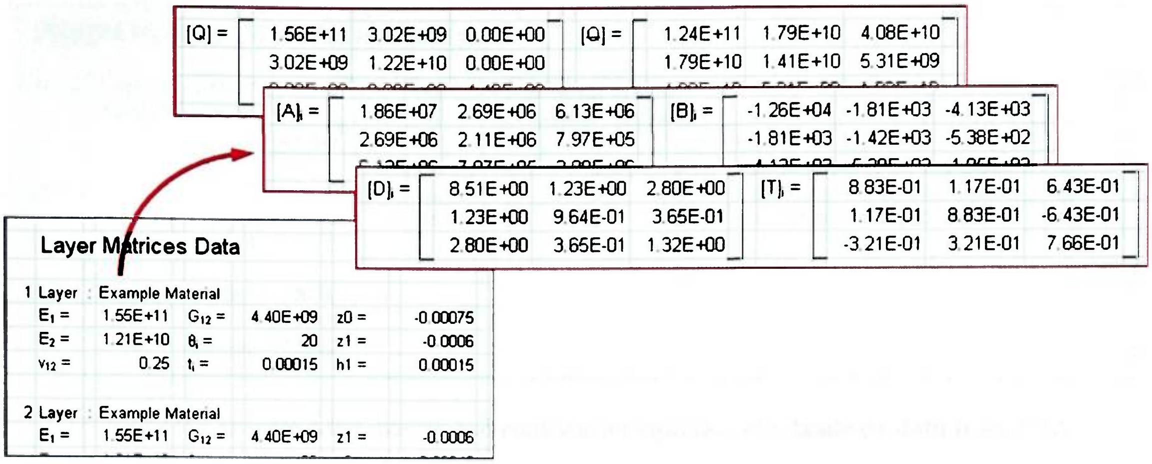

Figure 42: Example laminate matrices data from CFA.

With each layer's individual Q, $\bar{Q}$, A, B, D, and T matrices having been calculated in CFA's initial phase of execution, the stress-strain relationships of each layer can now be determined.

5.3.5 Laminate Stress Data

Having obtained crucial data from the previous section, the stress-strain relations for each layer of the laminate are needed in order to continue CFA's fiber-reinforced laminate failure analysis. Therefore, the sixth sheet of the CFA Excel® workbook labeled "STRESS DATA", shown in Figure 43, provides a detailed account of the stress-strain relationships of each layer within the laminate that is calculated in the next phase of CFA's laminate failure analysis.

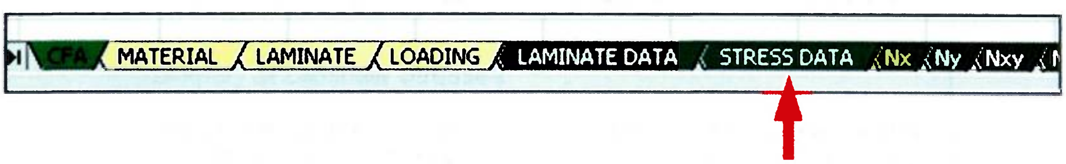

Figure 43: "STRESS DATA" sheet in CFA Excel® workbook.

This section of CFA's laminate failure analysis calculates the laminate ABD matrix and stress-strain relations, displays the constitutive equation of a laminate, and a summary of the principle material and global laminate stresses for each layer. Calculation of the principle material and global laminate stresses in each layer is essential for CFA to perform failure analysis using the several failure criteria presented earlier in this document.

An illustrative example of the ABD matrix and constitutive equation of a laminate, laminate stress-strain matrix data, and the summary of the principle material (1-2-3) and global laminate (x-y-z) stresses can bee seen in Figure 44, Figure 45, and Figure 46, respectively. Sample calculations of the ABD matrix and layer stresses are given in *Appendix B.1* and *Appendix B.2*, respectively.

[A][B][D] Matrices & Constitutive Equation

$$[A]=\begin{bmatrix}2.15E{+}08 & 13464747 & 0\\ 13464747 & 19373546 & 0\\ 0 & 0 & 15541310\end{bmatrix}$$

$$[B]=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$[D]=\begin{bmatrix}36.81434 & 4.13417 & 2.205098\\ 4.13417 & 3.837017 & 0.286983\\ 2.205098 & 0.286983 & 4.523432\end{bmatrix}$$

$$\begin{bmatrix}Nx\\ Ny\\ Nxy\\ Mx\\ My\\ Mxy\end{bmatrix}=\begin{bmatrix}2.15E{+}08 & 13464747 & 0 & 0 & 0 & 0\\ 13464747 & 19373546 & 0 & 0 & 0 & 0\\ 0 & 0 & 15541310 & 0 & 0 & 0\\ 0 & 0 & 0 & 36.81434 & 4.13417 & 2.205098\\ 0 & 0 & 0 & 4.13417 & 3.837017 & 0.286983\\ 0 & 0 & 0 & 2.205098 & 0.286983 & 4.523432\end{bmatrix}*\begin{bmatrix}\varepsilon_x^{o}\\ \varepsilon_y^{o}\\ \gamma_{xy}^{o}\\ \kappa_x^{o}\\ \kappa_y^{o}\\ \kappa_{xy}^{o}\end{bmatrix}$$

Figure 44: Example ABD matrix and constitutive equation of a laminate data from CFA.

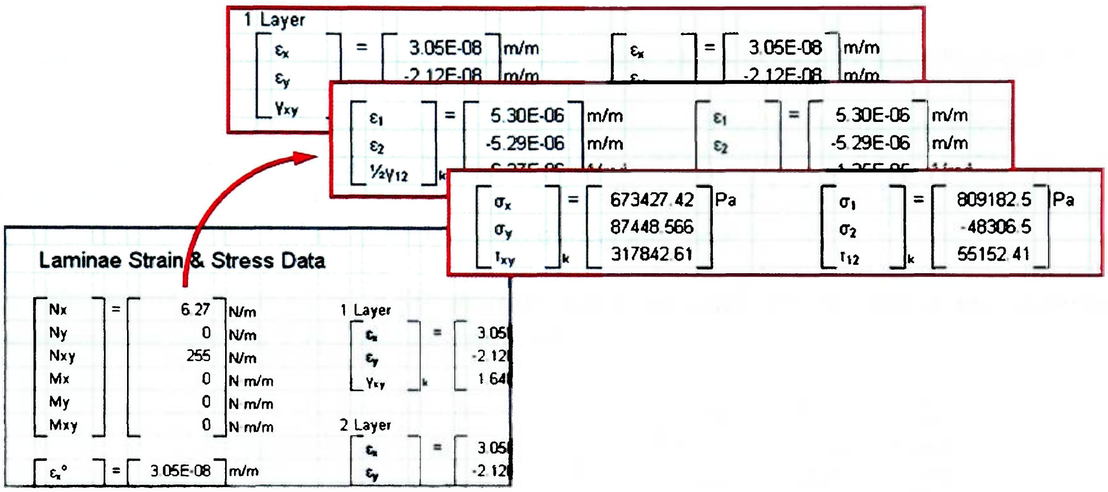

Figure 45: Example laminate stress-strain matrix data from CFA.

Summary of Laminate Stresses

Layer	σ_1 [Pa]	σ_2 [Pa]	τ_{12} [Pa]
1	809182.5	-48306.5	55152.41
2	-801645	48085.23	55445.3
3	4693.922	-166.03	72194.69
4	4693.922	-166.03	72194.69
5	4693.922	-166.03	72194.69
6	4693.922	-166.03	72194.69
7	4693.922	-166.03	72194.69
8	4693.922	-166.03	72194.69
9	-801645	48085.23	55445.3
10	809182.5	-48306.5	55152.41

Layer	σ_x [Pa]	σ_y [Pa]	τ_{xy} [Pa]
1	5.3E-06	-5.3E-06	1.25E-05
2	-5.2E-06	5.26E-06	1.26E-05
3	3.05E-08	-2.1E-08	1.64E-05
4	3.05E-08	-2.1E-08	1.64E-05
5	3.05E-08	-2.1E-08	1.64E-05
6	3.05E-08	-2.1E-08	1.64E-05
7	3.05E-08	-2.1E-08	1.64E-05
8	3.05E-08	-2.1E-08	1.64E-05
9	-5.2E-06	5.26E-06	1.26E-05
10	5.3E-06	-5.3E-06	1.25E-05

Figure 46: Example summary of principle (1-2-3) and global laminate (x-y-z) stresses from CFA.

After these calculations have been completed, in order to provide a means of 2D graphical analysis, CFA performs a stress coefficient analysis using these same methods of calculation presented above. CFA stress coefficient analysis is presented in the following section.

5.3.6 Laminate Stress Coefficient Data

Now that CFA has determined the stresses within each layer of the laminate, the failure analysis of the laminate can commence. However, before the failure analysis phase of CFA is developed, it is important to utilize the same calculation methods, which have been used thus far, to determine the laminate's stress coefficient data. Therefore, sheets seven through twelve of the CFA Excel® workbook labeled "Nx", "Ny", "Nxy", "Mx", "My", and "Mxy", shown in Figure 47, provide a detailed account of the stress-strain coefficient relationships of each layer within the.

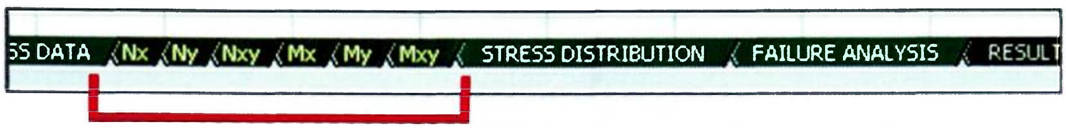

Figure 47: "Nx", "Ny", "Nxy", "Mx", "My", and "Mxy" sheets of the CFA Excel® workbook.

These data sheets in CFA are representative placeholders for a calculation where each of the force and moment resultants has been independently initialized to a value of 1. In doing so, stress coefficients are developed for the purposes of providing 2D graphical representations of the failure criterion used during analysis.

An illustrative example of independently equating each of the force and moment resultants to a value of 1 is given in Figure 48.

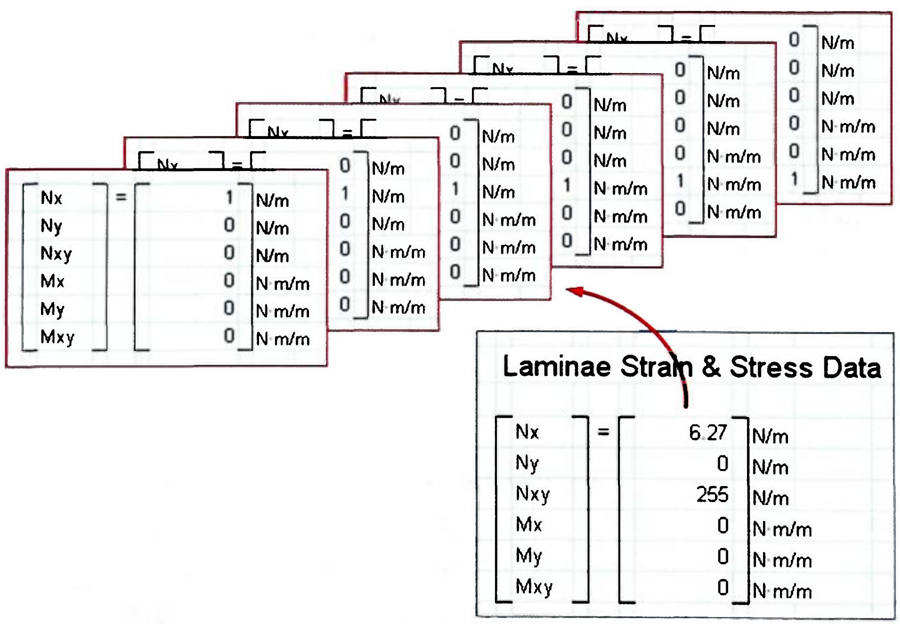

Figure 48: Example force and moment resultant setup for stress coefficient analysis from CFA.

The stress coefficients developed in this section are used for graphical 2D failure analysis which is developed in *Section 5.3.10.* Although CFA allows for only 2D illustrations of laminate failure envelopes, it is possible to create 3D representations using the same data.

5.3.7 Through Thickness Stress Distribution Data

With the use of the stress-strain data gathered in Section 5.3.5, CFA implements a 2D graphical analysis technique that allows the user to visualize the stress distribution through the thickness of the laminate. The through-thickness stress distribution of a laminate is developed in sheet thirteen of the CFA Excel® workbook labeled "STRESS DISTRIBUTION", shown in Figure 49.

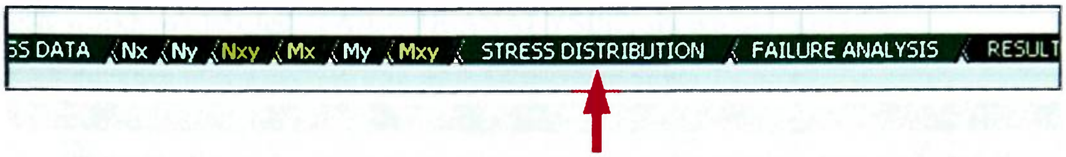

Figure 49: "STRESS DISTRIBUTION" sheet in CFA Excel® workbook.

CFA calculates the required stress distribution data for each layer of the laminate in order to generate a 2D representation of the laminate through-thickness stress distribution. An example illustration of the layer stress distribution data calculated in CFA can be seen in Figure 50 and a sample stress distribution calculation is given in *Appendix B.3*. Using this data, CFA generates the 2D stress distribution graph. An example is shown in Figure 51.

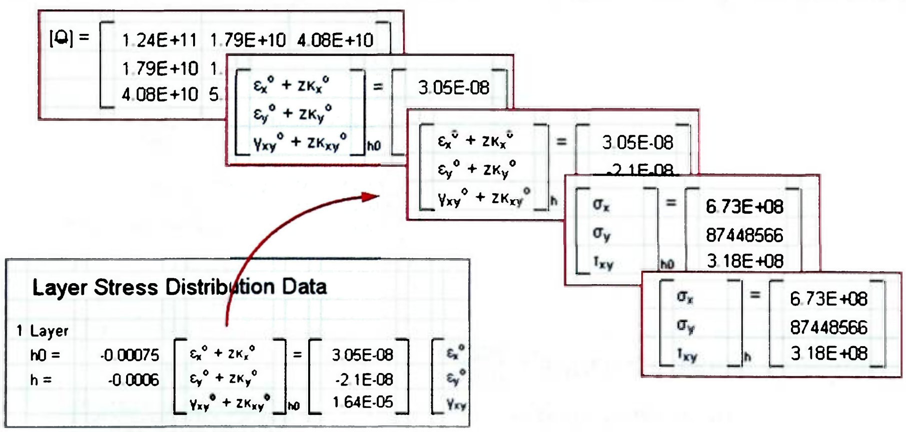

Figure 50: Example layer stress distribution data from CFA.

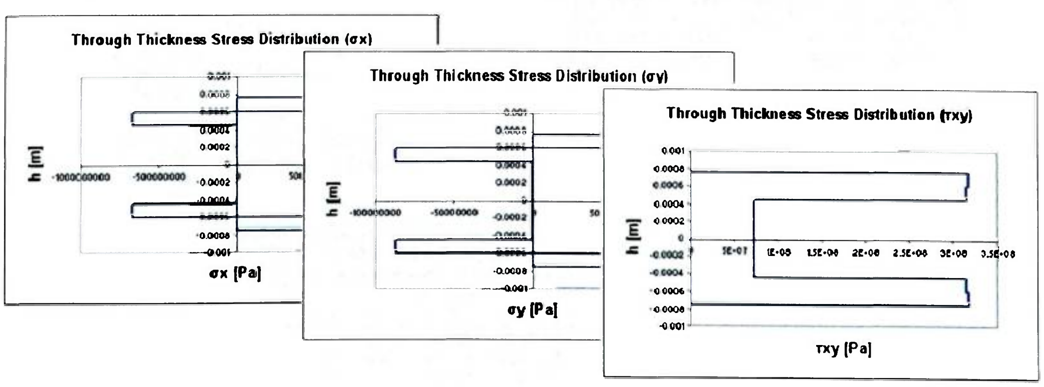

Figure 51: Example 2D laminate stress distribution representation from CFA.

Now that the stress-strain, stress distribution, and laminate matrices data have been calculated, CFA has acquired all of the tools needed in order to determine if laminate structural failure will occur.

5.3.8 Failure Criterion Analysis

Finally, CFA can perform laminate failure analysis using the various failure theories that were presented in *Section 4.2*. This analysis is contained in sheet fourteen of the CFA Excel® workbook labeled "FAILURE ANALYSIS", shown in Figure 52.

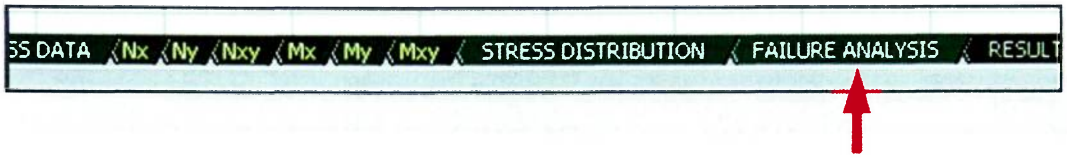

Figure 52: "FAILURE ANALYSIS" sheet in CFA Excel® workbook.

On this sheet in the CFA Excel® workbook, the data and results of failure analysis for each failure criterion are displayed. The user can independently verify that the laminate will not fail under each set of failure criterion. An illustrative representation of each failure criterion present within CFA's laminate failure analysis is given in Figure 53.

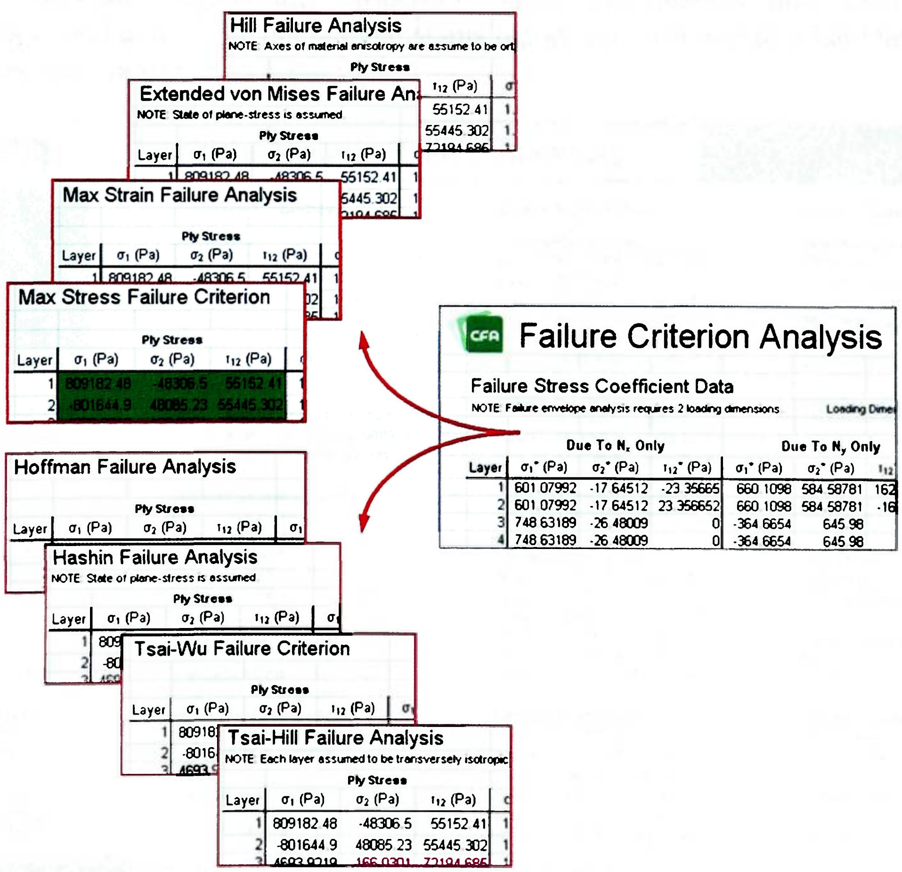

Figure 53: Representation of each failure criterion used within CFA's laminate failure analysis.

The failure analysis results obtained from CFA represent the culmination of the Excel® workbook's computational overhead. Now that CFA has finished its laminate failure analysis, a summary of the results is necessary and is presented in the subsequent section.

5.3.9 Laminate Analysis Results Summary

Since there is so much data collected throughout CFA's laminate failure analysis, it is necessary to provide the user with a summary of the results. This summary is contained in sheet fifteen of the CFA Excel® workbook labeled "RESULTS SUMMARY", shown in Figure 54.

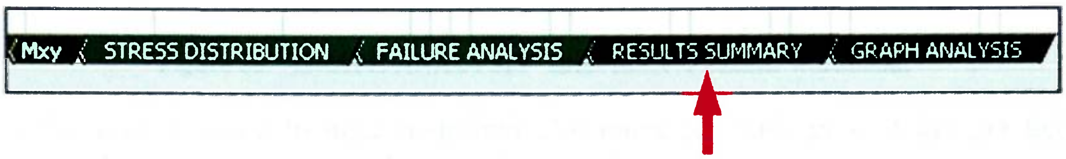

Figure 54: "FAILURE ANALYSIS" sheet in CFA Excel® workbook

This analysis summary provides the user with important information regarding whether or not the laminate has failed and from what failure criterion, the maximum and minimum principle and global laminate stresses and strains, the ABD matrix, the effective laminate engineering properties, layer deformation and total laminate deformation, and a laminate operational status report. An example of a laminate analysis results summary is shown in Figure 55.

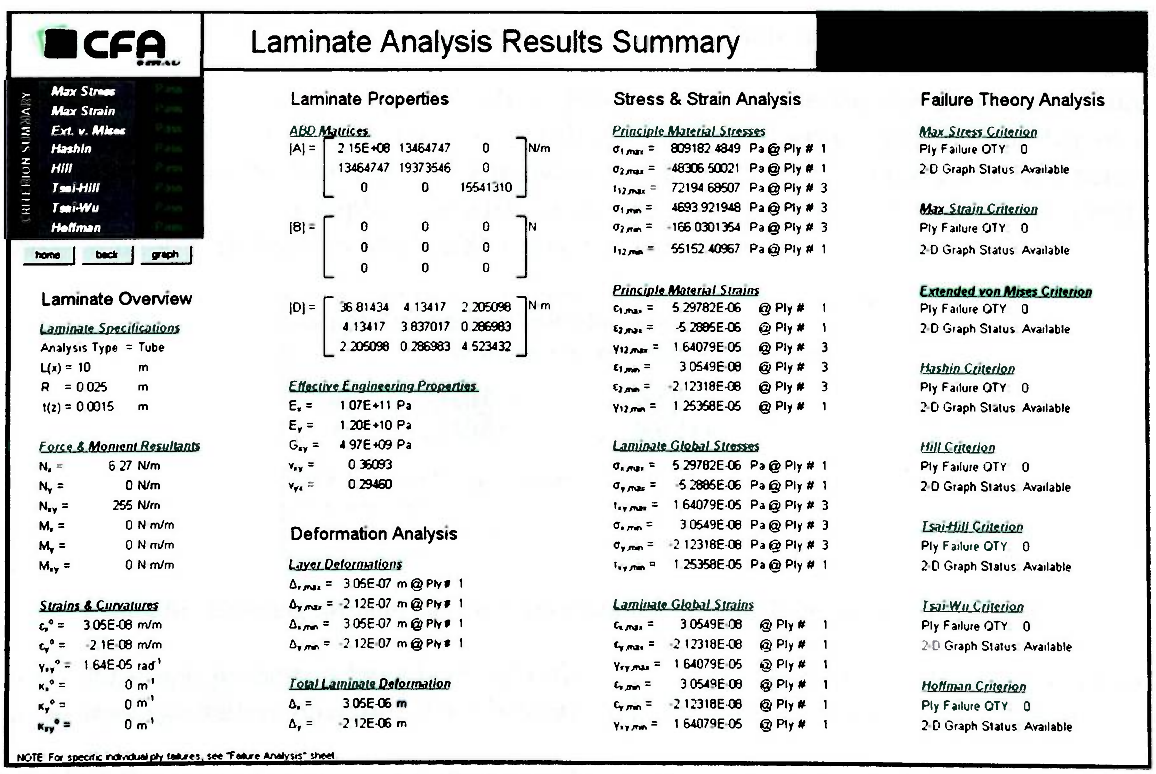

Figure 55: Example laminate analysis results summary from CFA.

5.3.10 Graph Analysis

A key feature of CFA is its ability to use MatLab® in order plot different failure envelopes for the laminate failure analysis that has been performed. The user defined properties required for graphical analysis in MatLab® are contained in the last sheet of the CFA Excel® workbook labeled "GRAPH ANALYSIS", shown in Figure 56.

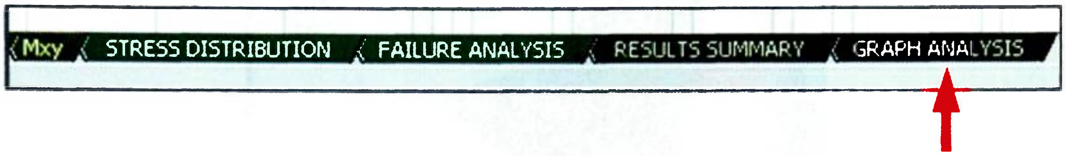

Figure 56: "GRAPH ANALYSIS" sheet in CFA Excel® workbook.

First, the user is asked to input a desired plot range for both axes of the 2D failure envelope. An example of this is shown in Figure 57.

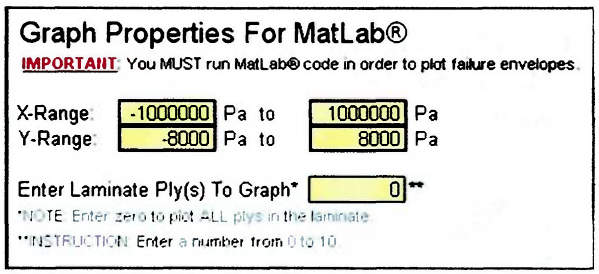

Figure 57: Entering the plot range for both axes of the 2D failure envelope in CFA.

Next, the user is asked to specify which laminate ply to use for the graphical failure envelope analysis. It is important to note that the user can only input the number of a valid layer within the laminate or the number zero in order to graph the entire laminate's failure envelope. An example of specifying the use of the entire laminate in the graphical analysis to be performed in MatLab® is given in Figure 58.

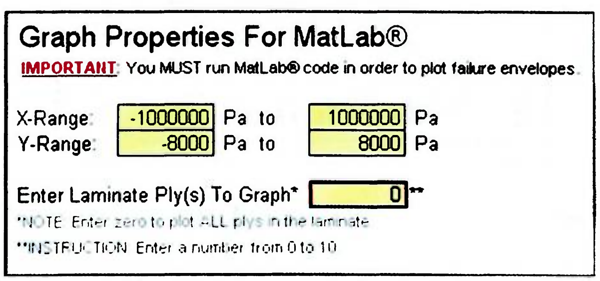

Figure 58: Entering the laminate layer specification for the 2D failure envelope in CFA.

After the graph properties have been specified for use in the graphical 2D representation of the laminate failure envelope, CFA's MatLab® software extension can be executed.

CFA's failure envelope representation was designed for use with MatLab® R2006a. The specific license and version that was used in this research is shown in Figure 59.

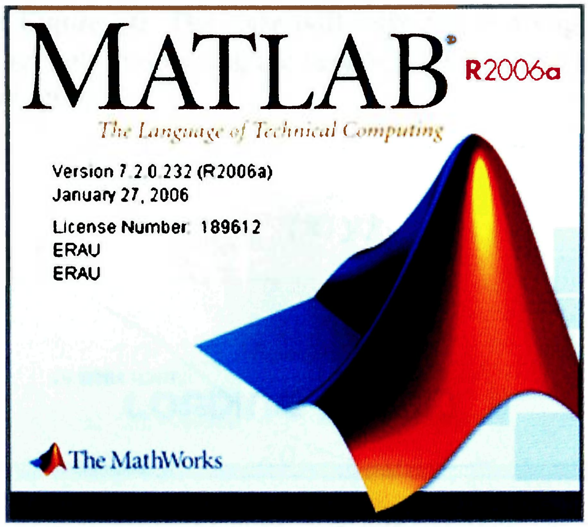

Figure 59: MatLab® license and version used in the development of CFA.

CFA's MatLab® software is a graphical user interface (GUI) that presents an intelligible method for laminate failure envelope analysis. This was created using the built-in GUI generator in MatLab®. An illustration of CFA's MatLab® software extension is shown in Figure 60.

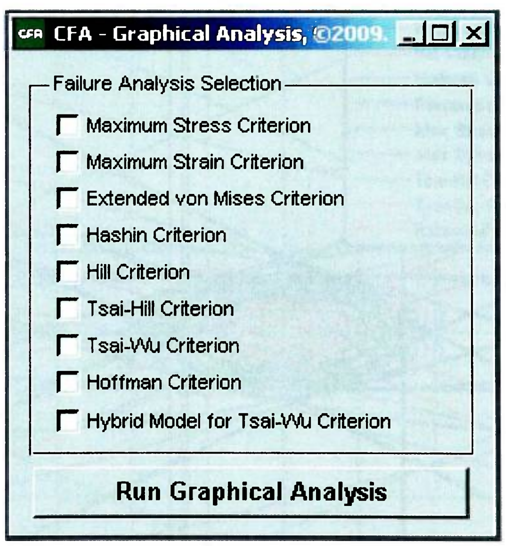

Figure 60: Illustration of CFA's MatLab® software extension GUI.

The 2D laminate failure envelope analysis performed in MatLab® by CFA is only applicable for conditions rendering a two dimensional analysis. This means that CFA's MatLab® software extension can only be executed when the sum of the non-zero force and moment resultants is equal to 2. If this is true then the user can perform a 2D graphical analysis in CFA to produce laminate failure envelopes.

Once the user has chosen which failure criteria to be used to determine the laminate failure envelope, the analysis is executed by clicking the "Run Graphical Analysis" button seen above, in Figure 60. The user will experience a slight duration of time in which the program is actively processing the results. During this time, a loading screen is displayed as shown in Figure 61.

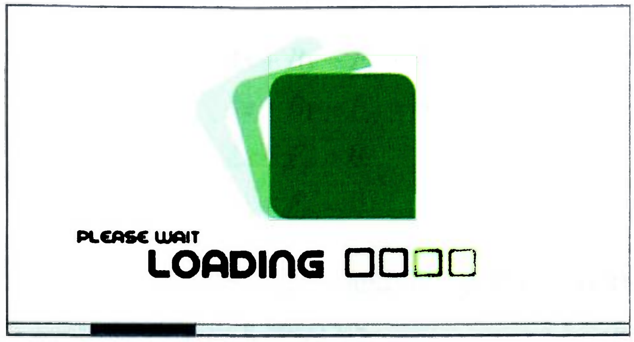

Figure 61: Loading screen displayed during laminate failure envelope analysis in CFA.

Finally, after MatLab® has finished processing CFA's laminate failure envelope analysis, a failure envelope is generated and displayed to the user. Using MatLab® commands, the user can alter or enhance the failure envelope figure as desired. An example failure envelope using all failure criteria is given in Figure 62.

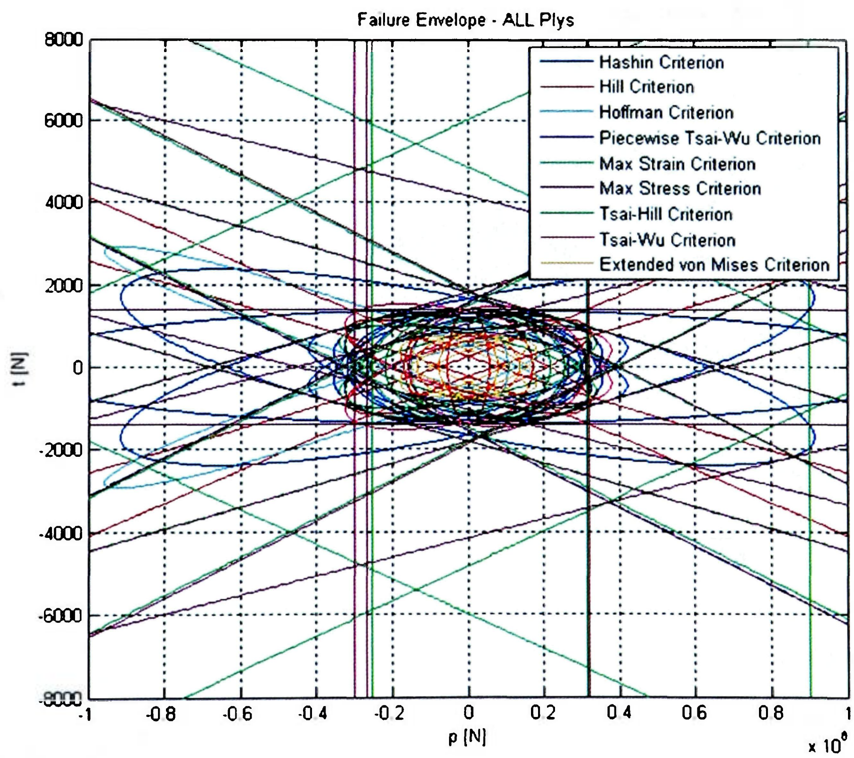

Figure 62: Example failure envelope incorporating all failure criteria from CFA.

The following sections specify how each failure criterion is graphed during the CFA laminate failure envelope analysis. Furthermore, the m-code used to generate CFA's MatLab® software extension GUI is given in *Appendix D.1*.

5.3.10.1 Graphing Maximum Stress Criterion in MatLab®

Using the definition presented in *Section 4.2.1*, the 2D graph of the Maximum Stress Failure Criterion is represented by a series of lines that are given by

$$\begin{aligned} Ax + By - F_T &= 0 \\ Ax + By - F_C &= 0 \\ Ax - F_T &= 0 \\ Ax - F_C &= 0 \end{aligned} \tag{5.1}$$

where A and B are the ply stress coefficients and F_T, and F_C are the ply failure stresses obtained from the CFA laminate failure analysis.

Using this linear failure relationship yields a 2D representation for a laminate failure envelope. An example of a Maximum Stress failure envelope from CFA is shown in Figure 63. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

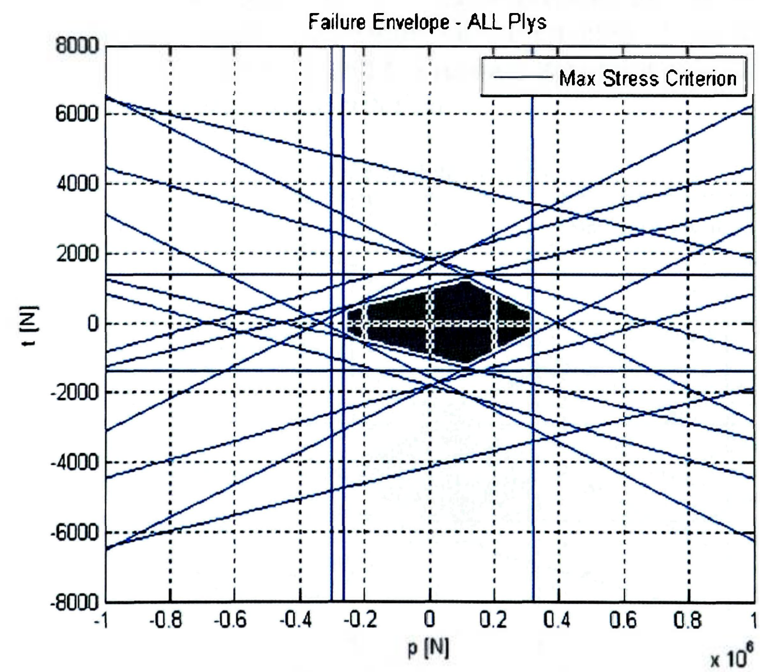

Figure 63: Example Maximum Stress Criterion failure envelope from CFA.

Furthermore, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.2*.

5.3.10.2 Graphing Maximum Strain Criterion in MatLab®

Using the definition presented in *Section 4.2.2*, the 2D graph of the Maximum Strain Failure Criterion is represented by a series of lines that are given by

$$Ax + By - C = 0 \tag{5.2}$$

where A, B, and C are coefficients that represent the 2D algebraic stress substitution solution from the Maximum Strain Criterion and are derived as

$$\begin{array}{lll} \text{Case 1} & \text{Case 2} & \text{Case 3} \\ A = \dfrac{A_1}{E_1} - \dfrac{A_2 \nu_{12}}{E_1} & A = \dfrac{A_2}{E_2} - \dfrac{A_1 \nu_{12}}{E_2} & A = A_3 \\ B = \dfrac{B_1}{E_1} - \dfrac{B_2 \nu_{12}}{E_1} & B = \dfrac{B_2}{E_2} - \dfrac{B_1 \nu_{12}}{E_2} & B = B_3 \\ C = \left(\varepsilon_1^T, \varepsilon_1^C\right) & C = \left(\varepsilon_2^T, \varepsilon_2^C\right) & C = G_{12}\left(\gamma_{12}^F, \gamma_{12}^{-F}\right) \end{array} \tag{5.3}$$

where A_1, A_2, A_3, B_1, B_2, and B_3 are the ply stress coefficients obtained from the CFA laminate failure analysis. This ply stress coefficient nomenclature will be used throughout the remainder of this thesis document.

Using this linear failure relationship yields a 2D representation for a laminate failure envelope. An example of a Maximum Strain failure envelope from CFA is shown in Figure 64. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

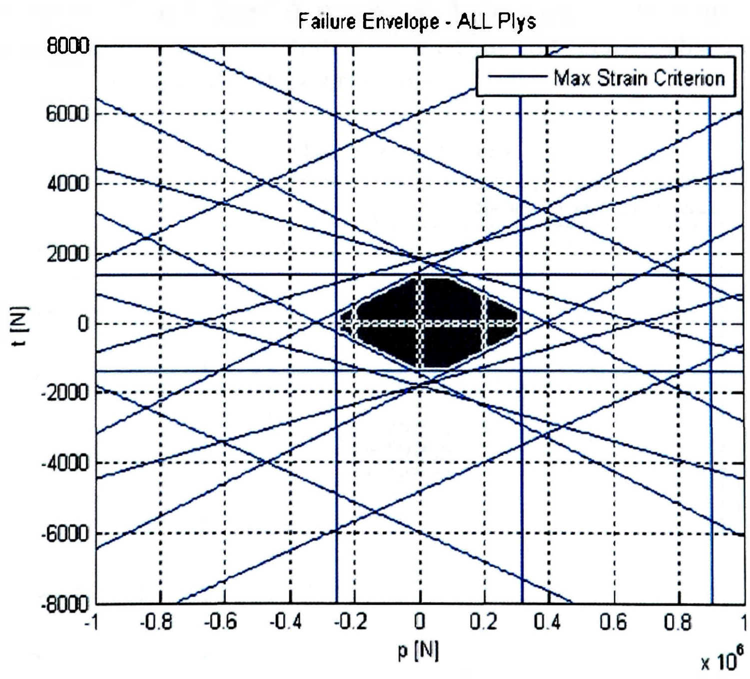

Figure 64: Example Maximum Strain Criterion failure envelope from CFA.

Moreover, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.3*.

5.3.10.3 Graphing Extended von Mises Criterion in MatLab®

Using the definition presented in *Section 4.2.3*, the 2D graph of the Extended von Mises Failure Criterion is represented by a series of ellipses that are given by

$$Ax^2 + By^2 + Cxy + D = 0 \tag{5.4}$$

where A, B, C, and D in this case are coefficients that represent the 2D algebraic stress substitution solution from the Extended von Mises Criterion and are derived as

$$\begin{aligned}
A &= FA_1^2 - 2FA_1A_2 + FA_2^2 + GA_2^2 + HA_1^2 + NA_3^2 \\
B &= FB_1^2 - 2FB_1B_2 + FB_2^2 + GB_2^2 + HB_1^2 + NB_3^2 \\
C &= 2FA_1B_1 - 2FA_1B_2 - 2FA_2B_1 + 2FA_2B_2 + 2GA_2B_2 + 2HA_1B_1 + 2NA_3B_3 \\
D &= -1
\end{aligned} \tag{5.5}$$

and

$$F = \frac{1}{2\left(\sigma_1^Y - \sigma_2^Y\right)^2} \qquad G = \frac{1}{2\left(\sigma_2^Y\right)^2} \qquad H = \frac{1}{2\left(\sigma_1^Y\right)^2} \qquad N = \frac{3}{\left(\tau_{12}^Y\right)^2} \tag{5.6}$$

Using this elliptic failure relationship yields a 2D representation for a laminate failure envelope. An example of an Extended von Mises failure envelope from CFA is shown in Figure 65. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

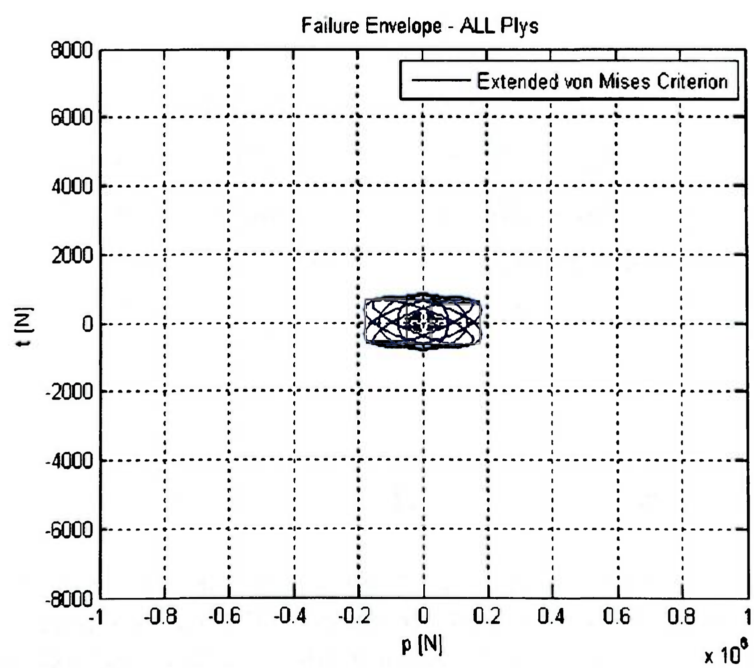

Figure 65: Example Extended von Mises Criterion failure envelope from CFA.

Additionally, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.4.*

5.3.10.4 Graphing Hashin Criterion in MatLab®

Using the definition presented in *Section 4.2.4*, the 2D graph of the Hashin Failure Criterion is represented by a combination of ellipses and lines that are given by

$$\begin{aligned} &Ax^2 + By^2 + Cxy + D = 0 \\ &Ax + By + C = 0 \end{aligned} \tag{5.7}$$

where A, B, C, and D in this case are coefficients that represent the 2D algebraic stress substitution solution from the Hashin Criterion and are derived as

$$\begin{array}{lll} \text{Elliptic Case 1} & \text{Elliptic Case 2} & \text{Linear Case} \\ A = FA_1^2 + GA_3^2 & A = HA_2^2 + GA_3^2 & A = NA_1 \\ B = FB_1^2 + GB_3^2 & B = HB_2^2 + GB_3^2 & B = NB_1 \\ C = 2FA_1B_1 + 2GA_3B_3 & C = 2HA_2B_2 + 2GA_3B_3 & C = -1 \\ D = -1 & D = -1 & \end{array} \tag{5.8}$$

with

$$F = \frac{1}{\left(\sigma_1^T\right)^2} \qquad G = \frac{1}{\left(\tau_{12}^F\right)^2} \qquad H = \frac{1}{\left(\sigma_2^T\right)^2} \qquad N = \frac{1}{\sigma_1^C} \tag{5.9}$$

and

$$\begin{array}{ll} \text{Elliptic Case 3} & \text{Elliptic Case 4} \\ A = FA_2^2 + GA_3^2 & A = HA_1^2 + GA_3^2 \\ B = FB_2^2 + GB_3^2 & B = HB_1^2 + GB_3^2 \\ C = 2FA_2B_2 + 2GA_3B_3 & C = 2HA_1B_1 + 2GA_3B_3 \\ D = -1 & D = -1 \end{array} \tag{5.10}$$

with

$$F = \frac{1}{\left(\sigma_2^C\right)^2} \qquad G = \frac{1}{\left(\tau_{12}^{-F}\right)^2} \qquad H = \frac{1}{\left(\sigma_1^C\right)^2} \tag{5.11}$$

Using these failure relationships yields a 2D representation for a laminate failure envelope. An example of a Hashin failure envelope from CFA is shown in Figure 66. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

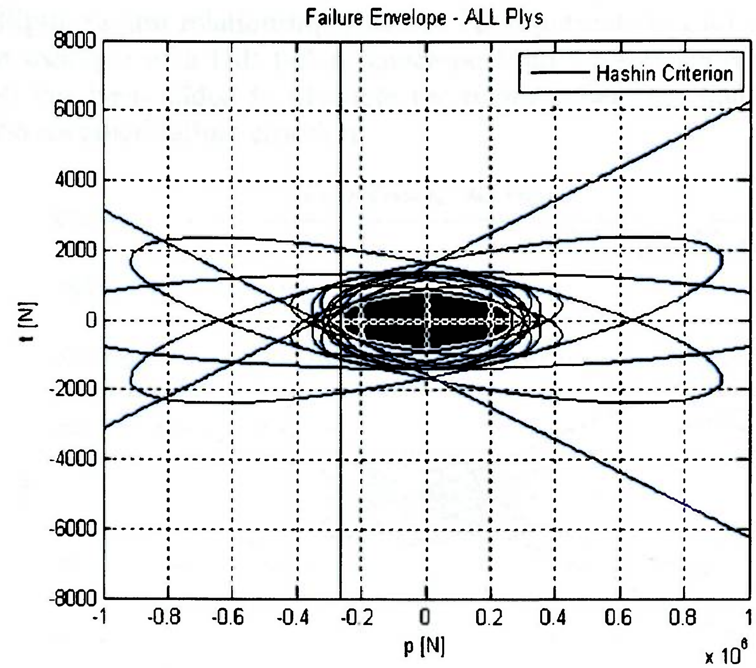

Figure 66: Example Hashin Criterion failure envelope from CFA.

Furthermore, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.5*.

5.3.10.5 Graphing Hill Criterion in MatLab®

Using the definition presented in *Section 4.2.5*, the 2D graph of the Hill Failure Criterion is represented by a series of ellipses that are given by

$$Ax^2 + By^2 + Cxy + D = 0 \tag{5.12}$$

where A, B, C, and D in this case are coefficients that represent the 2D algebraic stress substitution solution from the Hill Criterion and are derived as

$$\begin{aligned} A &= FA_2^2 + GA_1^2 + HA_1^2 - 2HA_1A_2 + HA_2^2 + 2NA_3^2 \\ B &= FB_2^2 + GB_1^2 + HB_1^2 - 2HB_1B_2 + HB_2^2 + 2NB_3^2 \\ C &= 2FA_2B_2 + 2GA_1B_1 + 2HA_1B_1 - 2HA_1B_2 - 2HA_2B_1 + 2HA_2B_2 + 4NA_3B_3 \\ D &= -1 \end{aligned} \tag{5.13}$$

and

$$F = \frac{1}{2}\left[\frac{1}{\left(\sigma_2^y\right)^2} - \frac{1}{\left(\sigma_1^y\right)^2}\right],\quad G = \frac{1}{2}\left[\frac{1}{\left(\sigma_1^y\right)^2} - \frac{1}{\left(\sigma_2^y\right)^2}\right],\quad H = \frac{1}{2}\left[\frac{1}{\left(\sigma_1^y\right)^2} + \frac{1}{\left(\sigma_2^y\right)^2}\right],\quad N = \frac{1}{2\left(\tau_{12}^y\right)^2} \tag{5.14}$$

Using this elliptic failure relationship yields a 2D representation for a laminate failure envelope. An example of a Hill failure envelope from CFA is shown in Figure 67. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

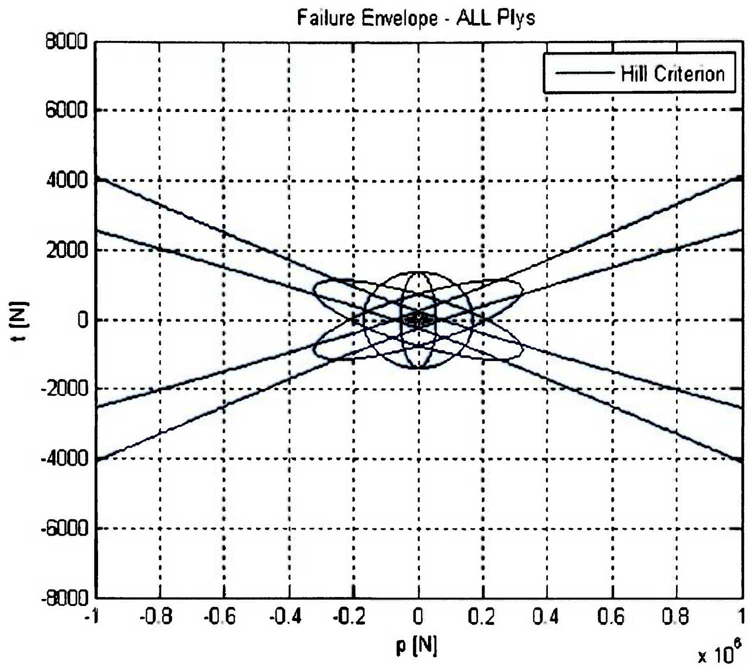

Figure 67: Example Hill Criterion failure envelope from CFA.

Moreover, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.6*.

5.3.10.6 Graphing Tsai-Hill Criterion in MatLab®

Using the definition presented in *Section 4.2.6*, the 2D graph of the Tsai-Hill Failure Criterion is represented by a series of ellipses that are given by

$$Ax^2 + By^2 + Cxy + D = 0 \tag{5.15}$$

where A, B, C, and D in this case are coefficients that represent the 2D algebraic stress substitution solution from the Tsai-Hill Criterion and are derived as

$$\begin{aligned} A &= \frac{A_1^2}{\left(\sigma_1^Y\right)^2} + \frac{A_2^2}{\left(\sigma_2^Y\right)^2} - \frac{A_1 A_2}{\left(\sigma_1^Y\right)^2} + \frac{A_3^2}{\left(\tau_{12}^Y\right)^2}, \quad B = \frac{B_1^2}{\left(\sigma_1^Y\right)^2} + \frac{B_2^2}{\left(\sigma_2^Y\right)^2} - \frac{B_1 B_2}{\left(\sigma_1^Y\right)^2} + \frac{B_3^2}{\left(\tau_{12}^Y\right)^2} \\ C &= \frac{2A_1B_1}{\left(\sigma_1^Y\right)^2} + \frac{2A_2B_2}{\left(\sigma_2^Y\right)^2} - \frac{A_1B_2}{\left(\sigma_1^Y\right)^2} - \frac{A_2B_1}{\left(\sigma_1^Y\right)^2} + \frac{2A_3B_3}{\left(\tau_{12}^Y\right)^2}, \quad D = -1 \end{aligned} \tag{5.16}$$

Using this elliptic failure relationship yields a 2D representation for a laminate failure envelope. An example of a Tsai-Hill failure envelope from CFA is shown in Figure 68. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

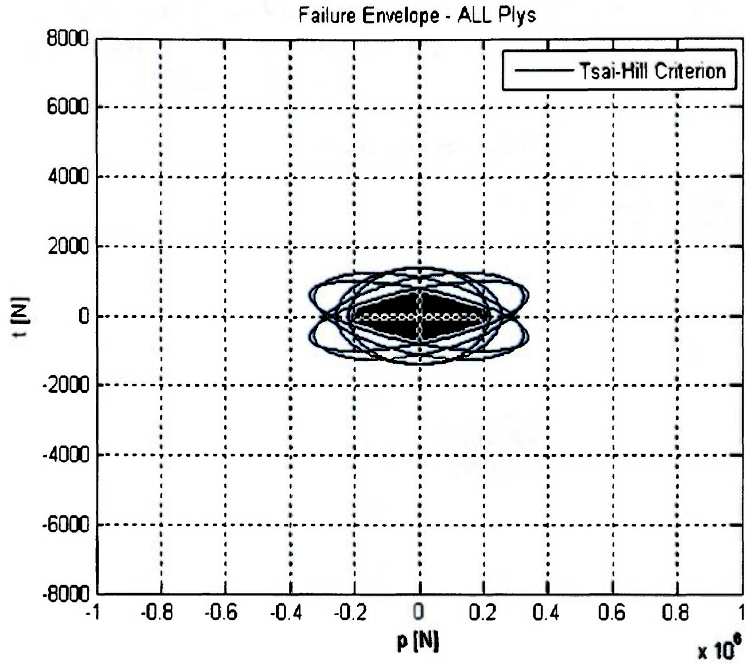

Figure 68: Example Tsai-Hill Criterion failure envelope from CFA.

Additionally, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.7*.

5.3.10.7 Graphing Tsai-Wu Criterion in MatLab®

Using the definition presented in *Section 4.2.7*, the 2D graph of the Tsai-Wu Failure Criterion is represented by a series of ellipses that are given by

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0 \tag{5.17}$$

where A, B, C, D, E, and F in this case are coefficients that represent the 2D algebraic stress substitution solution from the Tsai-Wu Criterion and are derived as

$$\begin{aligned}
&A = F_{11}A_1^2 + F_{22}A_2^2 + \left(-\sqrt{F_{11}F_{22}}\right)A_1A_2 + F_{66}A_3^2 \\
&B = F_{11}B_1^2 + F_{22}B_2^2 + \left(-\sqrt{F_{11}F_{22}}\right)B_1B_2 + F_{66}B_3^2 \\
&C = 2F_{11}A_1B_1 + 2F_{22}A_2B_2 + \left(-\sqrt{F_{11}F_{22}}\right)A_1B_2 + \left(-\sqrt{F_{11}F_{22}}\right)A_2B_1 + 2F_{66}A_3B_3 \\
&D = F_1A_1 + F_2A_2, \qquad E = F_1B_1 + F_2B_2, \qquad F = -1
\end{aligned} \tag{5.18}$$

and

$$F_1 = \frac{1}{\sigma_1^T} + \frac{1}{\sigma_1^C} \quad F_2 = \frac{1}{\sigma_2^T} + \frac{1}{\sigma_2^C} \quad F_{11} = \frac{1}{\sigma_1^T \sigma_1^C} \quad F_{22} = \frac{1}{\sigma_2^T \sigma_2^C} \quad F_{66} = \frac{1}{\left(\tau_{12}^F\right)^2} \tag{5.19}$$

Using this elliptic failure relationship yields a 2D representation for a laminate failure envelope. An example of a Tsai-Wu failure envelope from CFA is shown in Figure 69. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

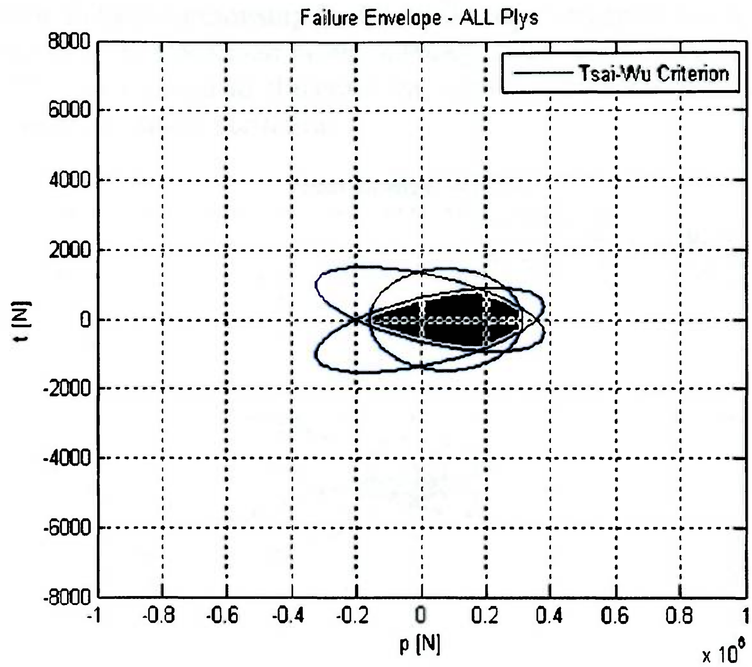

Figure 69: Example Tsai-Wu Criterion failure envelope from CFA.

Furthermore, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.8*.

5.3.10.8 Graphing Hoffman Criterion in MatLab®

Using the definition presented in *Section 4.2.8*, the 2D graph of the Hoffman Failure Criterion is represented by a series of ellipses that are given by

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0 \tag{5.20}$$

where A, B, C, D, E, and F in this case are coefficients that represent the 2D algebraic stress substitution solution from the Hoffman Criterion and are derived as

$$\begin{aligned} &A = C_1A_1^2 + C_2A_2^2 - C_3A_1A_2 + C_9A_3^2, \quad B = C_1B_1^2 + C_2B_2^2 - C_3B_1B_2 + C_9B_3^2 \\ &C = 2C_1A_1B_1 + 2C_2A_2B_2 - C_3A_1B_2 - C_3A_2B_1 + 2C_9A_3B_3, \quad D = C_4A_1 + C_5A_2 \\ &E = C_4B_1 + C_5B_2, \quad F = -1 \end{aligned} \tag{5.21}$$

and

$$C_1 = \frac{1}{\sigma_1^T \sigma_1^C}, \quad C_2 = \frac{1}{\sigma_2^T \sigma_2^C}, \quad C_3 = -\left(\frac{1}{\sigma_1^T \sigma_1^C} + \frac{1}{\sigma_2^T \sigma_2^C} \right)$$
$$C_4 = \frac{1}{\sigma_1^T} - \frac{1}{\sigma_1^C}, \quad C_5 = \frac{1}{\sigma_2^T} - \frac{1}{\sigma_2^C}, \quad C_9 = \frac{1}{\left(\tau_{12}^F\right)^2} \tag{5.22}$$

Using this elliptic failure relationship yields a 2D representation for a laminate failure envelope. An example of a Hoffman failure envelope from CFA is shown in Figure 70. A shaded region has been added to illustrate the region where the laminate will not fail based upon the specified failure criterion.

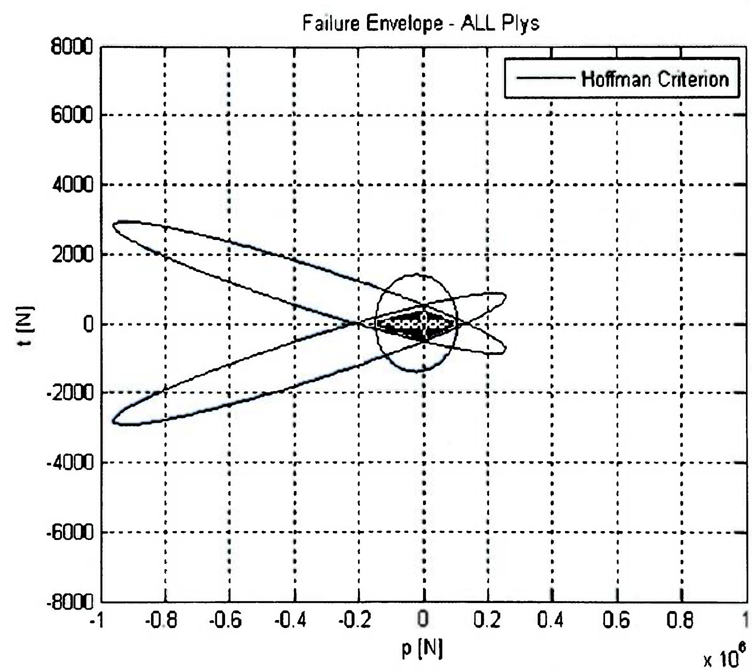

Figure 70: Example Hoffman Criterion failure envelope from CFA.

Moreover, the MatLab® m-code used by CFA to generate this failure envelope has been included in this research and is given in *Appendix D.9.*

5.3.10.9 Graphing the Piecewise Tsai-Wu Criterion Representation in MatLab®

As an added feature, CFA employs a method by which a piecewise representation of the Tsai-Wu Criterion can be produced. This method allows for the 2D failure envelope to be clearly conveyed. As a method that automatically generates the innermost area for a typical Tsai-Wu failure envelope, the usefulness of this type of analysis method is unmistakable.

Likewise, it can be shown, as seen in Figure 71, that a Piecewise Tsai-Wu failure envelope provides the user with a uniquely intelligible failure envelope representation.

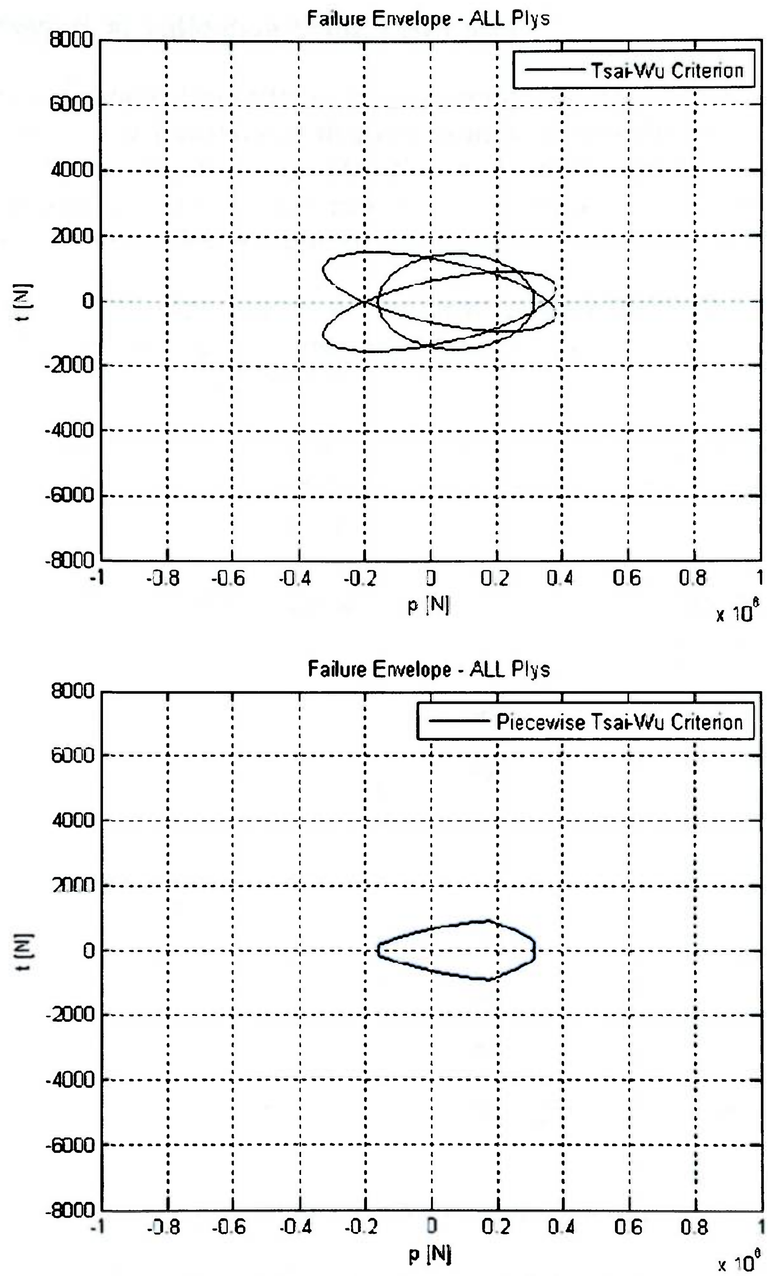

Figure 71: Comparison of a Tsai-Wu failure envelope with that of its piecewise representation.

Although CFA is capable of determining a piecewise representation for the Tsai-Wu Criterion, CFA lacks the ability to ascertain it's piecewise formulaic representation. Nonetheless, with its simplistic and comprehensible representation of a Tsai-Wu failure envelope, it is apparent that this method of failure envelope creation and analysis has immense potential.

Additionally, the MatLab® m-code used by CFA to generate a piecewise representation of the Tsai-Wu Failure Criterion has been included in this research and is given in *Appendix D.10.*

5.4 CFA Numerical Validation Using Test Cases

Since CFA was developed in order to provide computer-aided fiber-reinforced laminate failure analysis for the purposes of this research, it is possible that human error could have been introduced into its design. However, as with all software programs developed to provide engineering support and analysis, a multitude of software tests need to be performed in order to confirm its validity.

A series of twelve test cases were used in order to validate the CFA software. These test cases mostly included testing via numerical validation. However, most test cases included a graphical means of validation as well.

For illustrative purposes, a sample test case result is given in Figure 72 where it is evident that the fiber-reinforced laminate failure analysis conducted by CFA correlates with a similar analysis taken from *Reference 4*.

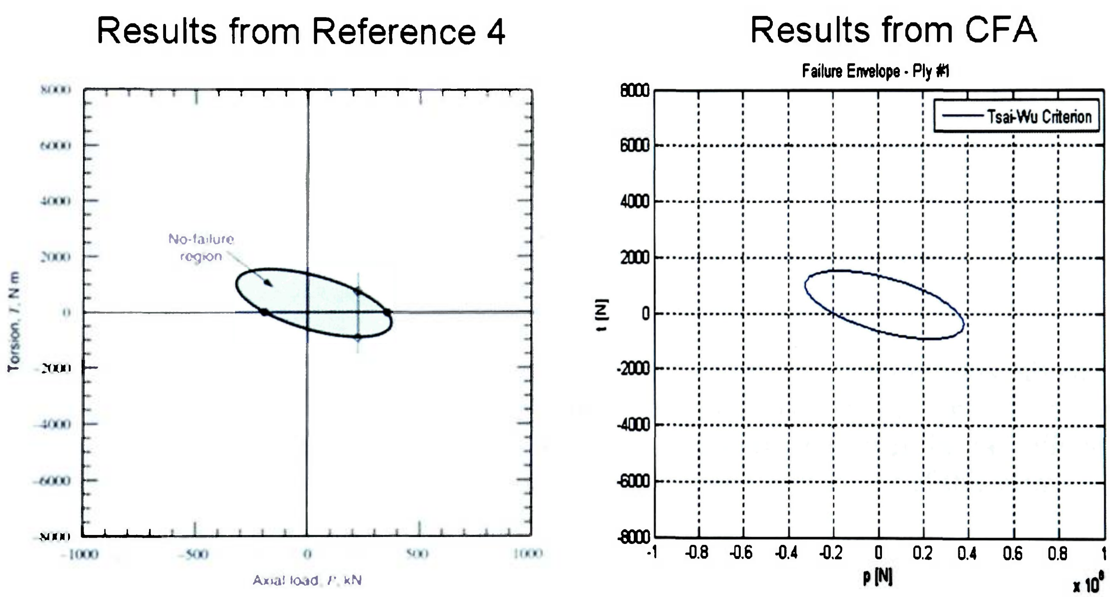

Figure 72: Sample CFA validation test case.

Furthermore, the CFA software validation test cases have been supplied in *Appendix C.1* through *Appendix C.12*.

CHAPTER 6

Conclusion

With the completion and validation of the CFA software, conclusions about its effectiveness, usability, and educational significance can be drawn. The concluding remarks about this research and the CFA software in general are given in the following sections. The CFA software has been included on the inside back cover of this print.

6.1 Conclusions about CFA

In conclusion, CFA exhibits an immense potential for the advancement of Embry-Riddle's educational prowess in the field of Composite Materials. With its ability to perform fiber-reinforced laminate failure analysis using the universal engineering software platforms of Excel® and MatLab®, CFA provides Embry-Riddle with a novel capability for future composite materials research.

Although fiber-reinforced laminate failure analysis is not a trivial topic, CFA was developed to provide a knowledgeable engineer with an intelligible software utensil for the design of fiber-reinforced laminates. CFA's simplistic exploitation of Excel® and MatLab® make it an indispensable tool for engineering education instruction.

The demand for exceptional engineering software is ever increasing. Because the CFA software will become a staple in Composite Materials education at Embry-Riddle, future research and development into CFA advancements and other material science engineering tools is essential.

6.2 Suggestions for Further Research

Although, the CFA software has its limitations, it provides a foundation for further research at Embry-Riddle Aeronautical University. With CFA being a new platform for educational development at Embry-Riddle, it is important to pioneer suggestions for further research using the CFA software. Therefore, the author suggests the following future research topics:

- Import CFA results into the composite design workbench of CATIA®. It is possible that the results of CFA's laminate failure analysis could be compared to the finite element analysis results of CATIA®.

- Expand the Piecewise Representation Method (PRM), used in CFA for the piecewise representation of the Tsai-Wu Criterion, to include the other failure criteria presented in this thesis document. The ability to capture a piecewise graphical and formulaic representation of a specific failure criterion can be supportive during fiber-reinforced laminate failure analysis.

- Determine if the failure criteria, presented in this thesis document, are dependent on composite laminate types, ply materials, ply orientations, and/or ply thicknesses. This would require extensive testing in order to hypothesize these correlations, if any.

- Develop a new failure criterion. With CFA's ability to provide quick analytical analyses of fiber-reinforced laminates, a new failure criterion can be developed in accordance with laminate specimen testing.

- Incorporate strain distribution and principle material stress distribution analyses in CFA. These types of analyses would compliment CFA's existing global laminate stress distribution analysis.

- Compile CFA into a self-sustaining software package that does not require the use of Excel® nor MatLab® to provide fiber-reinforced laminate failure analysis.

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APPENDIX A

Sample Material Properties

A.1 Sample Material Properties

Type	CFRP	BFRP	CFRP	GFRP	KFRP	CFRTP	CFRP	CFRP	CCRP	CCRP
Fiber	T300	B(4)	AS	E-glass	Kev 49	AS 4	H-IM6	T300	T300	T300
Matrix	N5208	N5505	3501	epoxy	epoxy.	PEEK	epoxy	F934	F934	F934
						APC2		4-mil	13-mil	7-mil
Engineering constants, GPa or dimensionless								tape	cloth	cloth
Ex	181 00	204 00	138 00	38.60	76 00	134.00	203.00	148.00	74 00	66.00
Ey	10.30	18 50	8.96	8.27	5.50	8.90	11.20	9.65	74.00	66.00
nu/x	0.28	0.23	0.30	0 26	0.34	0.28	0.32	0.30	0 05	0 04
Es	7.17	5 59	7.10	4.14	2.30	5.10	8.40	4.55	4.55	4.10
Vf	0.700	0 500	0.660	0.450	0 600	0 660	0.660	0.600	0 600	0.600
rho	1.600	2 000	1.600	1.800	1.460	1 600	1.600	1.500	1.500	1.500
ho, mm	0.125	0 125	0 125	0 125	0 125	0 125	0.125	0 100	0 325	0.175
Ply stiffness,GPa										
Qxx	181.81	204.98	138.81	39.17	76.64	134.70	204.14	148 87	74.19	66.13
Qyy	10.35	18.59	9.01	8 39	5.55	8.95	11.26	9.71	74.19	66.13
Qxy	2.90	4 28	2 70	2.18	1.89	2.51	3.58	2.91	3 71	2.91
Qss	7.17	5.59	7.10	4 14	2.30	5.10	8 40	4.55	4.55	4 10
Ply strength,MPa										
X	1500	1260	1447	1062	1400	2130	3500	1314	499	375
X'	1500	2500	1447	610	235	1100	1540	1220	352	279
Y	40	61	52	31	12	80	56	43	458	368
Y'	246	202	206	118	53	200	150	168	352	278
S	68	67	93	72	34	160	98	48	46	46
Fxy*	-0.5	-0.5	-0.5	-0 5	-0.5	-0 5	-0.5	-0 5	-0.5	-0 5

Figure 73: Sample material properties [7].

APPENDIX B

Sample Calculations

This appendix includes sample calculations for reference only. These calculations are similar to those required for laminate failure analysis and should be use used as examples only. The following calculations assume a $[\pm 20/0_3]_S$ laminate under loading conditions of

$$
\begin{aligned}
N_x &= 6.27 \frac{N}{m} \\
N_y &= 0 \frac{N}{m} \\
N_{xy} &= 255 \frac{N}{m} \\
M_x &= 0 \frac{N \cdot m}{m} \\
M_y &= 0 \frac{N \cdot m}{m} \\
M_{xy} &= 0 \frac{N \cdot m}{m}
\end{aligned}
\qquad \text{(b.1)}
$$

Through Classical Laminate Theory calculations, the resulting stresses in the first ply, with a fiber orientation of 20°, are

$$
\begin{aligned}
\sigma_1 &= 809182.48 Pa \\
\sigma_2 &= -48306.5 Pa \\
\tau_{12} &= 55152.41 Pa \\
\varepsilon_1 &= 5.298E-06 \\
\varepsilon_2 &= -2.06E-05 \\
\gamma_{12} &= 1.253E-05
\end{aligned}
\tag{b.2}
$$

with ply material properties of

$$
\begin{aligned}
t &= .150 mm \\
E_1 &= 155.00 GPa \\
E_2 &= 12.10 GPa \\
\nu_{12} &= 0.248 \\
G_{12} &= 4.40 GPa \\
\sigma_1^T &= 1.50E+09 Pa \\
\sigma_1^C &= -1.25E+09 Pa \\
\sigma_2^T &= 5.00E+07 Pa \\
\sigma_2^C &= -2.00E+08 Pa \\
\tau_{12}^F &= 1.00E+08 Pa \\
\tau_{12}^{-F} &= -1.00E+08 Pa
\end{aligned}
\tag{b.3}
$$

For the purpose of efficiency, CFA was used to generate the subsequent calculations.

B.1 Sample Layer ABD Contribution Calculation

For the specified 20° layer, the layer ABD contribution calculation is a crucial step necessary for performing fiber-reinforced laminate failure analysis. Using the definitions developed in *Section 3.6.6*, the layer ABD contributions are calculated as

$$
\begin{aligned}
[A]_k &= \begin{bmatrix} 1.86E+07 & 2.69E+06 & 6.13E+06 \\ 2.69E+06 & 2.11E+06 & 7.97E+05 \\ 6.13E+06 & 7.97E+05 & 2.90E+06 \end{bmatrix} N/m \\
[B]_k &= \begin{bmatrix} -1.26E+04 & -1.81E+03 & -4.13E+03 \\ -1.81E+03 & -1.42E+03 & -5.38E+02 \\ -4.13E+03 & -5.38E+02 & -1.95E+03 \end{bmatrix} N \\
[D]_k &= \begin{bmatrix} 8.51E+00 & 1.23E+00 & 2.80E+00 \\ 1.23E+00 & 9.64E-01 & 3.65E-01 \\ 2.80E+00 & 3.65E-01 & 1.32E+00 \end{bmatrix} Nm
\end{aligned}
\tag{b.4}
$$

where

$$
\begin{aligned}
[Q]_k &= \begin{bmatrix} 1.56E+11 & 3.02E+09 & 0.00E+00 \\ 3.02E+09 & 1.22E+10 & 0.00E+00 \\ 0.00E+00 & 0.00E+00 & 4.40E+09 \end{bmatrix} Pa \\
[\bar{Q}]_k &= \begin{bmatrix} 1.24E+11 & 1.79E+10 & 4.08E+10 \\ 1.79E+10 & 1.41E+10 & 5.31E+09 \\ 4.08E+10 & 5.31E+09 & 1.93E+10 \end{bmatrix} Pa \\
z_1 &= -0.00060m \\
z_0 &= -0.00075m
\end{aligned}
\tag{b.5}
$$

The summation of all layer ABD contributions results in the overall laminate ABD matrix, also known as the laminate stiffness matrix, discussed in *Section 3.6.6.*

B.2 Sample Layer Ply Stress Calculation

For the specified 20° layer, the layer stress calculation is another crucial step necessary for performing fiber-reinforced laminate failure analysis. Using the definitions developed in *Section 3.6*, the layer stresses are calculated as

$$
\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_l = \begin{bmatrix} 809182.5 \\ -48306.5 \\ 55152.41 \end{bmatrix} Pa \qquad \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_l = \begin{bmatrix} 673427.42 \\ 87448.566 \\ 317842.61 \end{bmatrix} Pa \tag{b.6}
$$

where

$$
\begin{aligned}
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_l &= \begin{bmatrix} 3.05E-08 \\ -2.12E-08 \\ 1.64E-05 \end{bmatrix} \\
[T]_l &= \begin{bmatrix} 8.83E-01 & 1.17E-01 & 6.43E-01 \\ 1.17E-01 & 8.83E-01 & -6.43E-01 \\ -3.21E-01 & 3.21E-01 & 7.66E-01 \end{bmatrix} \\
\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_l &= \begin{bmatrix} 5.30E-06 \\ -5.29E-06 \\ 1.25E-05 \end{bmatrix}
\end{aligned}
\tag{b.7}
$$

It is important to note that the multipliers used for engineering strain conversions have been accounted for. The layer stresses are then used for laminate failure analysis as can be seen through the failure criterion analyses given in *Appendices B.4* through *B.11*.

B.3 Sample Through-Thickness Stress Distribution

Once the individual ply stresses have been calculated for the specified laminate, given in Table 3, a through-thickness stress distribution can be created. Using x and y data points obtained from the ply stresses and their z thickness locations, result in a graphical distribution of stress throughout the thickness of the laminate as seen in Figure 74, Figure 75, and Figure 76.

Table 3: Sample laminate ply stresses.

Layer	σ_1 [Pa]	σ_2 [Pa]	τ_{12} [Pa]	σ_x [Pa]	σ_y [Pa]	τ_{xy} [Pa]
1	809182.5	-48306.5	55152.41	5.3E-06	-5.3E-06	1.25E-05
2	-801645	48085.23	55445.3	-5.2E-06	5.26E-06	1.26E-05
3	4693.922	-166.03	72194.69	3.05E-08	-2.1E-08	1.64E-05
4	4693.922	-166.03	72194.69	3.05E-08	-2.1E-08	1.64E-05
5	4693.922	-166.03	72194.69	3.05E-08	-2.1E-08	1.64E-05
6	4693.922	-166.03	72194.69	3.05E-08	-2.1E-08	1.64E-05
7	4693.922	-166.03	72194.69	3.05E-08	-2.1E-08	1.64E-05
8	4693.922	-166.03	72194.69	3.05E-08	-2.1E-08	1.64E-05
9	-801645	48085.23	55445.3	-5.2E-06	5.26E-06	1.26E-05
10	809182.5	-48306.5	55152.41	5.3E-06	-5.3E-06	1.25E-05

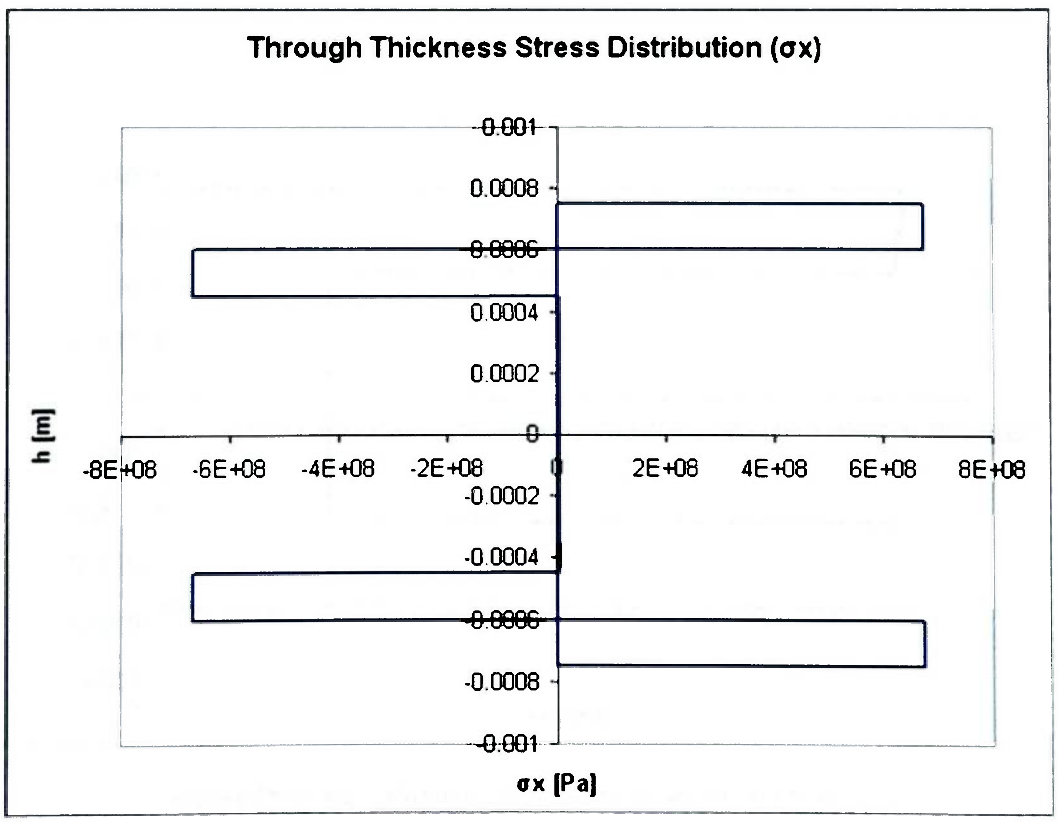

Figure 74: Sample through-thickness stress distribution for σ_x.

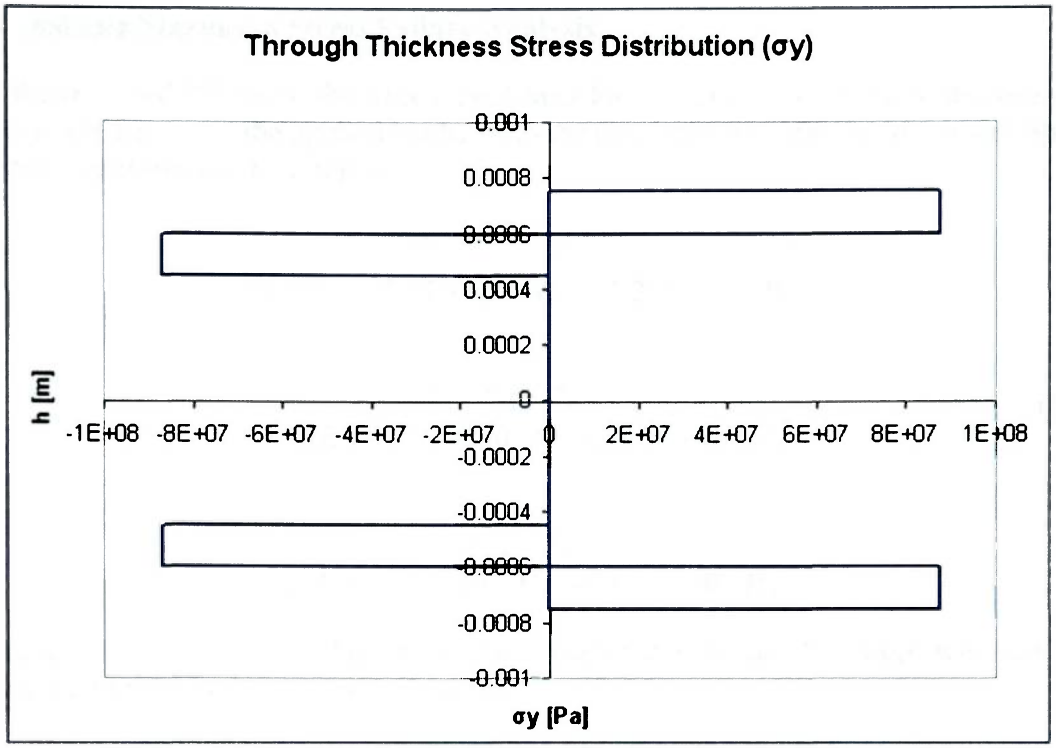

Figure 75: Sample through-thickness stress distribution for σ_y.

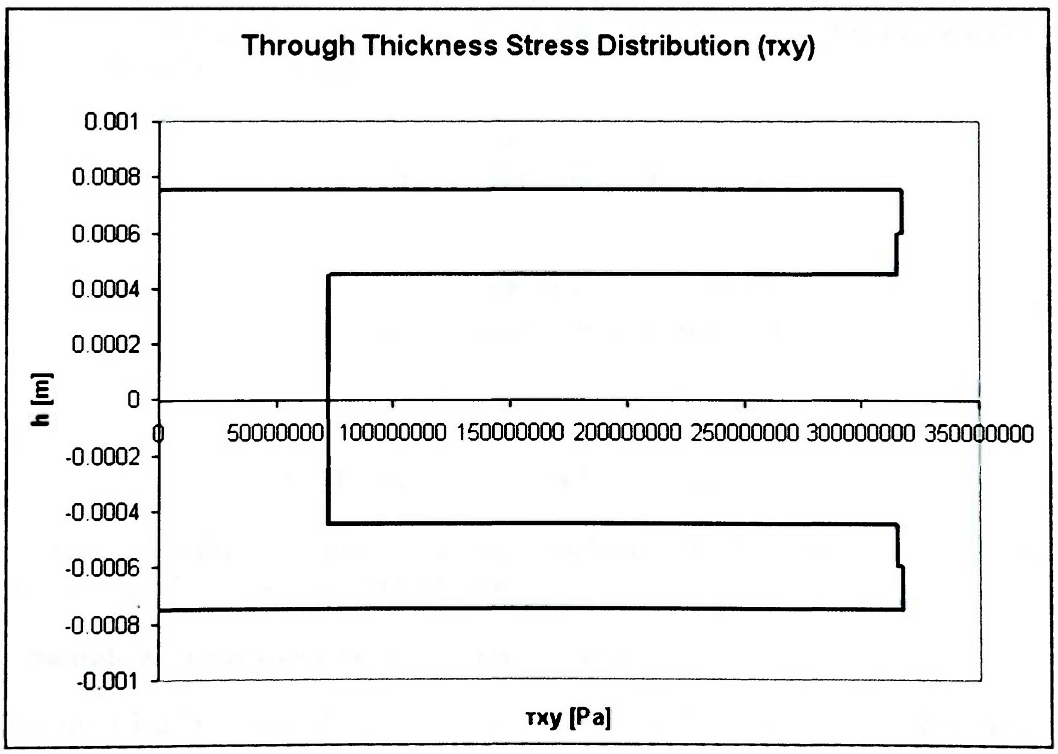

Figure 76: Sample through-thickness stress distribution for τ_{xy}.

B.4 Sample Maximum Stress Failure Analysis

For the specified 20° layer, the Maximum Stress Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Maximum Stress Criterion relations can be setup as

$$\begin{gathered}
\sigma_1^T > \sigma_1 > \sigma_1^C \\
1.50E+09 > 809182.48 > -1.25E+09 \ \ Pa \\
\\
\sigma_2^T > \sigma_2 > \sigma_2^C \\
5.00E+07 > -48306.5 > -2.00E+08 \ \ Pa \\
\\
\tau_{12}^F > \tau_{12} > \tau_{12}^{-F} \\
1.00E+08 > 55152.41 > -1.00E+08 \ \ Pa
\end{gathered} \tag{b.8}$$

Since each inequality is valid, it can be concluded that the 20° layer will not fail according to the Maximum Stress Criterion.

B.5 Sample Maximum Strain Failure Analysis

For the specified 20° layer, the Maximum Strain Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply strains, the Maximum Strain Criterion relations can be setup as

$$\begin{gathered}
\varepsilon_1^T > \varepsilon_1 > \varepsilon_1^C \\
9.60E-03 > 5.298E-06 > -7.74E-03 \\
\\
\varepsilon_2^T > \varepsilon_2 > \varepsilon_2^C \\
2.66E-02 > -2.06E-05 > -9.09E-03 \\
\\
\gamma_{12}^F > \gamma_{12} > \gamma_{12}^{-F} \\
2.27E-02 > 1.253E-05 > -2.27E-02
\end{gathered} \tag{b.9}$$

Since each inequality is valid, it can be concluded that the 20° layer will not fail according to the Maximum Strain Criterion.

B.6 Sample Extended von Mises Failure Analysis

For the specified 20° layer, the Extended von Mises Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Extended von Mises Criterion relations can be setup as

$$\frac{(\sigma_1-\sigma_2)^2}{2\sigma_1^T\sigma_2^T}+\frac{\sigma_2^2}{2(\sigma_2^T)^2}+\frac{\sigma_1^2}{2(\sigma_1^T)^2}+\frac{3\tau_{12}^2}{(\tau_{12}^F)^2}<1$$

$$\left(\begin{array}{l}\dfrac{(809182.48Pa-(-48306.5Pa))^2}{2(1.50E+09Pa)(5.00E+07Pa)}+\dfrac{(-48306.5Pa)^2}{2(5.00E+07Pa)^2}+\\ \dfrac{(809182.48Pa)^2}{2(1.50E+09Pa)^2}+\dfrac{3(55152.41Pa)^2}{(1.00E+08Pa)^2}\end{array}\right)<1$$

$$\frac{(\sigma_1-\sigma_2)^2}{2\sigma_1^C\sigma_2^C}+\frac{\sigma_2^2}{2(\sigma_2^C)^2}+\frac{\sigma_1^2}{2(\sigma_1^C)^2}+\frac{3\tau_{12}^2}{(\tau_{12}^{-F})^2}<1$$

$$\left(\begin{array}{l}\dfrac{(809182.48Pa-(-48306.5Pa))^2}{2(-1.25E+09Pa)(-2.00E+08Pa)}+\dfrac{(-48306.5Pa)^2}{2(-2.00E+08Pa)^2}+\\ \dfrac{(809182.48Pa)^2}{2(-1.25E+09Pa)^2}+\dfrac{3(55152.41Pa)^2}{(-1.00E+08Pa)^2}\end{array}\right)<1 \qquad \text{(b.10)}$$

Since each inequality is valid, it can be concluded that the 20° layer will not fail according to the Extended von Mises Criterion.

B.7 Sample Hashin Failure Analysis

For the specified 20° layer, the Hashin Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Hashin Criterion relations can be setup as

$$M^T=\left(\frac{\sigma_2}{\sigma_2^T}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^F}\right)^2<1 \qquad M^C=\left(\frac{\sigma_2}{\sigma_2^C}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^{-F}}\right)^2<1$$

$$\left(\frac{-48306.5Pa}{5.00E+07Pa}\right)^2+\left(\frac{55152.41Pa}{1.00E+08Pa}\right)^2<1 \qquad \left(\frac{-48306.5Pa}{-2.00E+08Pa}\right)^2+\left(\frac{55152.41Pa}{-1.00E+08Pa}\right)^2<1$$

$$F^T=\left(\frac{\sigma_1}{\sigma_1^T}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^F}\right)^2<1 \qquad F^C=\left(\frac{\sigma_1}{\sigma_1^C}\right)^2<1 \qquad \text{(b.11)}$$

$$\left(\frac{809182.48Pa}{1.50E+09Pa}\right)^2+\left(\frac{55152.41Pa}{1.00E+08Pa}\right)^2<1 \qquad \left(\frac{809182.48Pa}{-1.25E+09Pa}\right)^2<1$$

$$FMS^C=\left(\frac{\sigma_1}{\sigma_1^C}\right)^2+\left(\frac{\tau_{12}}{\tau_{12}^{-F}}\right)^2<1$$

$$\left(\frac{809182.48Pa}{-1.25E+09Pa}\right)^2+\left(\frac{55152.41Pa}{-1.00E+08Pa}\right)^2<1$$

Since each inequality is valid, it can be concluded that the 20° layer will not fail according to the Hashin Criterion. This type of failure analysis can indicate which type of failure mode has occurred.

B.8 Sample Hill Failure Analysis

For the specified 20° layer, the Hill Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Hill Criterion relations can be setup as

$$\left[\frac{\sigma_2^2}{2(\sigma_2^T)^2}-\frac{\sigma_2^2}{2(\sigma_1^T)^2}\right]+\left[\frac{\sigma_1^2}{2(\sigma_1^T)^2}-\frac{\sigma_1^2}{2(\sigma_2^T)^2}\right]+\left[\frac{(\sigma_1-\sigma_2)^2}{2(\sigma_1^T)^2}+\frac{(\sigma_1-\sigma_2)^2}{2(\sigma_2^T)^2}\right]+\frac{\tau_{12}^2}{(\tau_{12}^F)^2}<1$$

$$\left(\begin{array}{l}\left[\frac{(-48306.5Pa)^2}{2(5.00E+07Pa)^2}-\frac{(-48306.5Pa)^2}{2(1.50E+09Pa)^2}\right]+\\ \left[\frac{(809182.48Pa)^2}{2(1.50E+09Pa)^2}-\frac{(809182.48Pa)^2}{2(5.00E+07Pa)^2}\right]+\\ \left[\frac{(809182.48Pa-(-48306.5Pa))^2}{2(1.50E+09Pa)^2}+\frac{(809182.48Pa-(-48306.5Pa))^2}{2(5.00E+07Pa)^2}\right]+\\ \frac{(55152.41Pa)^2}{(1.00E+08Pa)^2}\end{array}\right)<1$$

$$\left[\frac{\sigma_2^2}{2(\sigma_2^C)^2}-\frac{\sigma_2^2}{2(\sigma_1^C)^2}\right]+\left[\frac{\sigma_1^2}{2(\sigma_1^C)^2}-\frac{\sigma_1^2}{2(\sigma_2^C)^2}\right]+\left[\frac{(\sigma_1-\sigma_2)^2}{2(\sigma_1^C)^2}+\frac{(\sigma_1-\sigma_2)^2}{2(\sigma_2^C)^2}\right]+\frac{\tau_{12}^2}{(\tau_{12}^{-F})^2}<1$$

$$\left(\begin{array}{l}\left[\frac{(-48306.5Pa)^2}{2(-2.00E+08Pa)^2}-\frac{(-48306.5Pa)^2}{2(-1.25E+09Pa)^2}\right]+\\ \left[\frac{(809182.48Pa)^2}{2(-1.25E+09Pa)^2}-\frac{(809182.48Pa)^2}{2(-2.00E+08Pa)^2}\right]+\\ \left[\frac{(809182.48Pa-(-48306.5Pa))^2}{2(-1.25E+09Pa)^2}+\frac{(809182.48Pa-(-48306.5Pa))^2}{2(-2.00E+08Pa)^2}\right]+\\ \frac{(55152.41Pa)^2}{(-1.00E+08Pa)^2}\end{array}\right)<1 \qquad \text{(b.12)}$$

Since each inequality is valid, it can be concluded that the 20° layer will not fail according to the Hill Criterion.

B.9 Sample Tsai-Hill Failure Analysis

For the specified 20° layer, the Tsai-Hill Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Tsai-Hill Criterion relations can be setup as

$$\left(\frac{\sigma_1}{\sigma_1^T}\right)^2+\left(\frac{\sigma_2}{\sigma_2^T}\right)^2-\frac{\sigma_1\sigma_2}{\left(\sigma_1^T\right)^2}+\left(\frac{\tau_{12}}{\tau_{12}^F}\right)^2<1$$

$$\left(\begin{matrix}\left(\frac{809182.48Pa}{1.50E+09Pa}\right)^2+\left(\frac{-48306.5Pa}{5.00E+07Pa}\right)^2-\\ \frac{(809182.48Pa)(-48306.5Pa)}{(1.50E+09Pa)^2}+\left(\frac{1.253E-05}{1.00E+08Pa}\right)^2\end{matrix}\right)<1 \qquad \text{(b.13)}$$

$$\left(\frac{\sigma_1}{\sigma_1^C}\right)^2+\left(\frac{\sigma_2}{\sigma_2^C}\right)^2-\frac{\sigma_1\sigma_2}{\left(\sigma_1^C\right)^2}+\left(\frac{\tau_{12}}{\tau_{12}^{-F}}\right)^2<1$$

$$\left(\begin{matrix}\left(\frac{809182.48Pa}{-1.25E+09Pa}\right)^2+\left(\frac{-48306.5Pa}{-2.00E+08Pa}\right)^2-\\ \frac{(809182.48Pa)(-48306.5Pa)}{(-1.25E+09Pa)^2}+\left(\frac{1.253E-05}{-1.00E+08Pa}\right)^2\end{matrix}\right)<1$$

Since each inequality is valid, it can be concluded that the 20° layer will not fail according to the Tsai-Hill Criterion.

B.10 Sample Tsai-Wu Failure Analysis

For the specified 20° layer, the Tsai-Wu Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Tsai-Wu Criterion relation can be setup as

$$\sigma_1\left[\frac{1}{\sigma_1^T}+\frac{1}{\sigma_1^C}\right]+\sigma_2\left[\frac{1}{\sigma_2^T}+\frac{1}{\sigma_2^C}\right]+\frac{\sigma_1^2}{\sigma_1^T\sigma_1^C}+\frac{\sigma_2^2}{\sigma_2^T\sigma_2^C}+\frac{\tau_{12}^2}{\left(\tau_{12}^F\right)^2}-\sqrt{\left(\frac{1}{\sigma_1^T\sigma_1^C\sigma_2^T\sigma_2^C}\right)}\sigma_1\sigma_2<1$$

$$\left(\begin{matrix}\left[\frac{809182.48Pa}{1.50E+09Pa}+\frac{809182.48Pa}{-1.25E+09Pa}\right]+\left[\frac{-48306.5Pa}{5.00E+07Pa}+\frac{-48306.5Pa}{-2.00E+08Pa}\right]+\\ \frac{(809182.48Pa)^2}{(1.50E+09Pa)(-1.25E+09Pa)}+\frac{(-48306.5Pa)^2}{(5.00E+07Pa)(-2.00E+08Pa)}-\\ \sqrt{\left(\frac{(809182.48Pa)^2(-48306.5Pa)^2}{(1.5E+9Pa)(-1.25E+9Pa)(5.E+7Pa)(-2.E+8Pa)}\right)}+\frac{(55152.41Pa)^2}{(1.00E+8Pa)^2}\end{matrix}\right)<1 \qquad \text{(b.14)}$$

Since the inequality is valid, it can be concluded that the 20° layer will not fail according to the Tsai-Wu Criterion.

B.11 Sample Hoffman Failure Analysis

For the specified 20° layer, the Hoffman Failure Criterion is used to determine if the ply will fail under the applied loads. With the given ply stresses, the Hoffman Criterion relations can be setup as

$$\frac{\sigma_1^2}{\sigma_1^T\sigma_1^C}+\frac{\sigma_2^2}{\sigma_2^T\sigma_2^C}+\left(\frac{\sigma_1\sigma_2}{\sigma_1^T\sigma_1^C}+\frac{\sigma_1\sigma_2}{\sigma_2^T\sigma_2^C}\right)+\left(\frac{\sigma_1}{\sigma_1^T}-\frac{\sigma_1}{\sigma_1^C}\right)+\left(\frac{\sigma_2}{\sigma_2^T}-\frac{\sigma_2}{\sigma_2^C}\right)+\frac{\tau_{12}^2}{\left(\tau_{12}^F\right)^2}<1$$

$$\left(\begin{array}{l}\dfrac{(809182.48Pa)^2}{(1.50E+09Pa)(-1.25E+09Pa)}+\dfrac{(-48306.5Pa)^2}{(5.00E+07Pa)(-2.00E+08Pa)}+\\ \left(\dfrac{(809182.48Pa)(-48306.5Pa)}{(1.50E+09Pa)(-1.25E+09Pa)}+\dfrac{(809182.48Pa)(-48306.5Pa)}{(5.00E+07Pa)(-2.00E+08Pa)}\right)+\\ \left(\dfrac{809182.48Pa}{1.50E+09Pa}-\dfrac{809182.48Pa}{-1.25E+09Pa}\right)+\left(\dfrac{-48306.5Pa}{5.00E+07Pa}-\dfrac{-48306.5Pa}{-2.00E+08Pa}\right)+\\ \dfrac{(55152.41Pa)^2}{(1.00E+08Pa)^2}\end{array}\right)<1 \qquad \text{(b.15)}$$

Since the inequality is valid, it can be concluded that the 20° layer will not fail according to the Hoffman Criterion.

APPENDIX C

CFA – Sample Validation Reports

This appendix provides a detailed account of twelve test cases used for validation of the CFA software. Each test case utilizes the following material properties for each layer of the specified laminates. These material properties are

$$
\begin{aligned}
t &= .150mm \\
E_1 &= 155.00GPa \\
E_2 &= 12.10GPa \\
\nu_{12} &= 0.248 \\
G_{12} &= 4.40GPa \\
\sigma_1^T &= 1.50E+09Pa \\
\sigma_1^C &= -1.25E+09Pa \\
\sigma_2^T &= 5.00E+07Pa \\
\sigma_2^C &= -2.00E+08Pa \\
\tau_{12}^F &= 1.00E+08Pa \\
\tau_{12}^{-F} &= -1.00E+08Pa
\end{aligned}
\tag{c.1}
$$

For the purposes of this research, the specified quantity of test cases is assumed adequate.

C.1 Case 1: Global Laminate Stress Distribution Numerical Validation

This numerical validation case is intended to corroborate CFA's global laminate stress distribution algorithms and illustrations with those provided from *Reference 4.*

A $[\pm 30/0]_S$ laminate is subjected to force and moment resultants of [4]

$$\begin{aligned} N_x &= 102400 N/m \\ N_y &= 18940 N/m \\ N_{xy} &= 0 \\ M_x &= 0 \\ M_y &= 0 \\ M_{xy} &= 0 \end{aligned} \tag{c.2}$$

The strains are determined to be [4]

$$\begin{aligned} \varepsilon_x &= 1000 \times 10^{-6} \\ \varepsilon_y &= 0 \\ \gamma_{xy} &= 0 \end{aligned} \tag{c.3}$$

When the force and moment resultants of the given above are entered into CFA, the resulting strains are shown in Figure 77.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k = \begin{bmatrix} 1.00E-03 \\ -3.21E-07 \\ 0.00E+00 \end{bmatrix} \begin{matrix} m/m \\ m/m \\ 1/rad \end{matrix}$$

Figure 77: Case 1 - CFA strain results.

It is important to note that since CFA uses Excel® for mathematical operations, the values obtained are not rounded. This is likely responsible for the insignificant value of ε_y that is calculated by CFA, given in Figure 77.

Continuing to use the reference example detailed in *Reference 4*, the stresses in the +30° layers of the laminate are [4]

$$\begin{aligned} \sigma_x &= 92.8 MPa \\ \sigma_y &= 30.1 MPa \\ \tau_{xy} &= 46.7 MPa \end{aligned} \tag{c.4}$$

with the stresses in the -30° layers being [4]

$$
\begin{aligned}
\sigma_x &= 92.8MPa \\
\sigma_y &= 30.1MPa \\
\tau_{xy} &= -46.7MPa
\end{aligned}
\tag{c.5}
$$

and the stresses in the 0° layers at [4]

$$
\begin{aligned}
\sigma_x &= 155.7MPa \\
\sigma_y &= 3.02MPa \\
\tau_{xy} &= 0MPa
\end{aligned}
\tag{c.6}
$$

Using CFA, the stresses in each layer are calculated and are shown in Figure 78.

Layer	σ_x [Pa]	σ_y [Pa]	τ_{xy} [Pa]
1	92791363	30060943	46702552
2	92791363	30060943	-4.7E+07
3	1.56E+08	3011448	0
4	1.56E+08	3011448	0
5	92791363	30060943	-4.7E+07
6	92791363	30060943	46702552

Figure 78: Case 1 - CFA resulting layer stresses.

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. Once the stresses are determined, the graphical stress distribution analysis is performed. For comparison, the stress distributions obtained from *Reference 4* and CFA are given in Figure 79 and Figure 80, respectively.

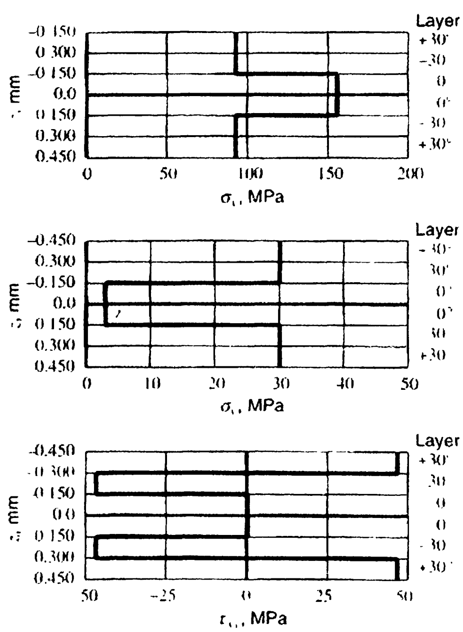

Figure 79: Case 1 – Reference stress distribution [4].

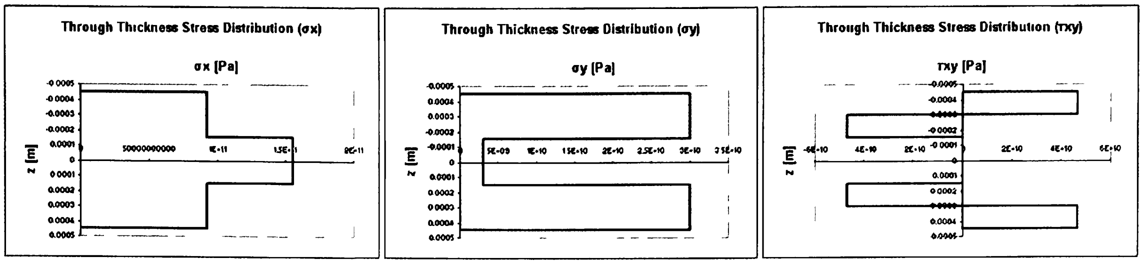

Figure 80: Case 1 – CFA's stress distribution.

Please note that the loading conditions specified above are to be assumed if any of the subsequent validation test cases do not specify otherwise. Also, the analysis presented in this section is used as substantiation for the algorithms and graphing techniques used in the CFA software.

C.2 Case 2: Laminate Stiffness [ABD] Matrix Numerical Validation

This numerical validation case is intended to corroborate CFA's laminate stiffness [ABD] matrix with that provided from *Reference 4.* Taken from the reference example, the [A], [B], and [D] matrices for the specified $[\pm 30/0]_S$ laminate are given as [4]

$$[A]=\begin{bmatrix}102.4\times10^6 & 18.94\times10^6 & 0\\ 18.94\times10^6 & 16.25\times10^6 & 0\\ 0 & 0 & 20.2\times10^6\end{bmatrix} N/m \tag{c.7}$$

$$[B]=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} N \tag{c.8}$$

$$[D]=\begin{bmatrix}5.78 & 1.766 & 1.261\\ 1.766 & 1.256 & 0.418\\ 1.261 & 0.418 & 1.850\end{bmatrix} N\cdot m \tag{c.9}$$

with the full constitutive equation of a laminate written as [4]

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\\ M_x\\ M_y\\ M_{xy}\end{Bmatrix}=\begin{bmatrix}102.4\times10^6 & 18.94\times10^6 & 0 & 0 & 0 & 0\\ 18.94\times10^6 & 16.25\times10^6 & 0 & 0 & 0 & 0\\ 0 & 0 & 20.2\times10^6 & 0 & 0 & 0\\ 0 & 0 & 0 & 5.78 & 1.766 & 1.261\\ 0 & 0 & 0 & 1.766 & 1.256 & 0.418\\ 0 & 0 & 0 & 1.261 & 0.418 & 1.850\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\\ \kappa_x^0\\ \kappa_y^0\\ \kappa_{xy}^0\end{Bmatrix} \tag{c.10}$$

Using CFA, the [A], [B], and [D] matrices are calculated and given in Figure 81.

$$[A] = \begin{bmatrix} 1.02E{+}08 & 18944756 & 0 \\ 18944756 & 16249944 & 0 \\ 0 & 0 & 20191006 \end{bmatrix} \text{N/m}$$

$$[B] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{N}$$

$$[D] = \begin{bmatrix} 5.77916 & 1.765701 & 1.261072 \\ 1.765701 & 1.256093 & 0.41767 \\ 1.261072 & 0.41767 & 1.849823 \end{bmatrix} \text{N·m}$$

Figure 81: Case 2 - Resulting [A], [B], and [D] matrices from CFA.

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.3 Case 3: Laminate Strains and Curvatures Numerical Validation

This numerical validation case is intended to corroborate CFA's laminate strains and curvatures calculations with that provided from *Reference 4*. Taken from the reference example, the reference strains and curvatures for the specified $[\pm30/0]_S$ laminate are given as [4]

$$\begin{aligned} \varepsilon_x^0 &= 1000\times10^{-6} & \kappa_x^0 &= 0 \\ \varepsilon_y^0 &= 0 & \kappa_y^0 &= 0 \\ \gamma_{xy}^0 &= 0 & \kappa_{xy}^0 &= 0 \end{aligned} \qquad (c.11)$$

When the force and moment resultants given above are entered into CFA, the resulting strains are shown in Figure 82.

$$\begin{bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{bmatrix} = \begin{bmatrix} 0.001 \\ -3.2E{-}07 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} \text{m/m} \\ \text{m/m} \\ \text{rad}^{-1} \\ \text{m}^{-1} \\ \text{m}^{-1} \\ \text{m}^{-1} \end{array}$$

Figure 82: Case 3 – CFA's reference strains and curvatures results.

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.4 Case 4: Max Stress Criterion (Axially Loaded Tube) Numerical Validation

This numerical validation case is intended to corroborate CFA's Maximum Stress Criterion calculations for an axially loaded tube with that provided from *Reference 4.*

A $[\pm 20/0_3]_S$ tube is subjected to force and moment resultants of [4]

$$
\begin{aligned}
N_x &= \frac{P}{2\pi R} = 6.37 N/m \\
N_y &= 0 \\
N_{xy} &= 0 \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned}
\tag{c.12}
$$

The summary of loads P (MN) to cause failure based upon the Maximum Stress Criterion is given in Figure 83, from the reference example.

Layer	Failure mode					
	σ_1^C	σ_1^T	σ_2^C	σ_2^T	$-\tau_{12}^F$	$+\tau_{12}^F$
+20	−0.327	+0.392	+1.780	−0.445	+0.673	−0.673
−20	−0.327	+0.392	+1.780	0.445	−0.673	+0.673
0	−0.262	+0.315	+1.186	−0.297	−∞	+∞

Figure 83: Case 4 – Reference loads to cause failure [4].

From CFA, the equivalent summary of loads to cause failure based upon the Maximum Stress Criterion is given in Figure 84.

Layer	Loads P [N] To Cause Failure For Specified Failure Mode					
	σ_1^T	σ_1^C	σ_2^T	σ_2^C	τ_{12}^F	$-\tau_{12}^F$
1	391759.57	-326466.3	-444842.3	1779369 1	-672124.9	672124 88
2	391759.57	-326466.3	-444842.3	1779369.1	672124.88	-672124.9
3	314545.52	-262121.3	-296422.5	1185690.1	∞	-∞
4	314545.52	-262121.3	-296422.5	1185690.1	∞	-∞
5	314545.52	-262121.3	-296422.5	1185690.1	∞	-∞
6	314545.52	-262121.3	-296422.5	1185690.1	∞	-∞
7	314545.52	-262121.3	-296422.5	1185690.1	∞	-∞
8	314545.52	-262121.3	-296422.5	1185690 1	∞	-∞
9	391759.57	-326466.3	-444842.3	1779369.1	672124.88	-672124.9
10	391759.57	-326466.3	-444842.3	1779369.1	-672124.9	672124.88

Figure 84: Case 4 – CFA's calculated loads to cause axial tube failure.

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.5 Case 5: Max Stress Criterion (Tube in Torsion) Numerical Validation

This numerical validation case is intended to corroborate CFA's Maximum Stress Criterion calculations for a torsionally loaded tube with that provided from *Reference 4.*

A $[\pm 20/0_3]_S$ tube is subjected to force and moment resultants of [4]

$$
\begin{aligned}
N_x &= \frac{P}{2\pi R} = 1432394 N/m \\
N_y &= 0 \\
N_{xy} &= \frac{T}{2\pi R^2} = 255 N/m \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned}
\qquad (c.13)
$$

The summary of loads T (MNm) to cause failure based upon the Maximum Stress Criterion is given in Figure 85, from the reference example.

Layer	Failure mode					
	σ_1^T	$-\sigma_1^C$	σ_2^T	$-\sigma_2^C$	$-\tau_{12}^F$	τ_{12}^F
-20	2620	-795	-3630	-1563	1205	-2420
20	-2620	-795	-3630	+1563	2420	+1205
0	[illegible]	[illegible]	[illegible]	[illegible]	1387	±1387

Figure 85: Case 5 – Reference loads to cause failure [4].

From CFA, the equivalent summary of loads to cause failure based upon the Maximum Stress Criterion is given in Figure 86.

Layer	Loads T [Nm] To Cause Failure For Specified Failure Mode					
	σ_1^T	σ_1^C	σ_2^T	σ_2^C	τ_{12}^F	$-\tau_{12}^F$
1	793.40185	-2620.992	-1561.851	3625.316	2413.3579	-1203.353
2	-793.4019	2620.9925	1561.851	-3625.316	1203.3535	-2413.358
3	∞	-∞	∞	-∞	1385.1435	-1385.144
4	∞	-∞	∞	-∞	1385.1435	-1385.144
5	∞	-∞	∞	-∞	1385.1435	-1385.144
6	∞	-∞	∞	-∞	1385.1435	-1385.144
7	∞	-∞	∞	-∞	1385.1435	-1385.144
8	∞	-∞	∞	-∞	1385.1435	-1385.144
9	-793.4019	2620.9925	1561.851	-3625.316	1203.3535	-2413.358
10	793.40185	-2620.992	-1561.851	3625.316	2413.3579	-1203.353

Figure 86: Case 5 – CFA's calculated loads to cause torsional tube failure.

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.6 Case 6: Max Stress Criterion (Tube w/Combined Load) Graphical Validation

This graphical validation case is intended to corroborate CFA's Maximum Stress Criterion calculations for a tube with a combined axial and torsional loading with that provided from *Reference 4*.

Using a $[\pm 20/0_3]_S$ tube subjected to force and moment resultants of [4]

$$
\begin{aligned}
N_x &= \frac{P}{2\pi R} = 6.37 N/m \\
N_y &= 0 \\
N_{xy} &= \frac{T}{2\pi R^2} = 255 N/m \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned}
\tag{c.14}
$$

an illustration of their graphical comparison is given in Figure 87.

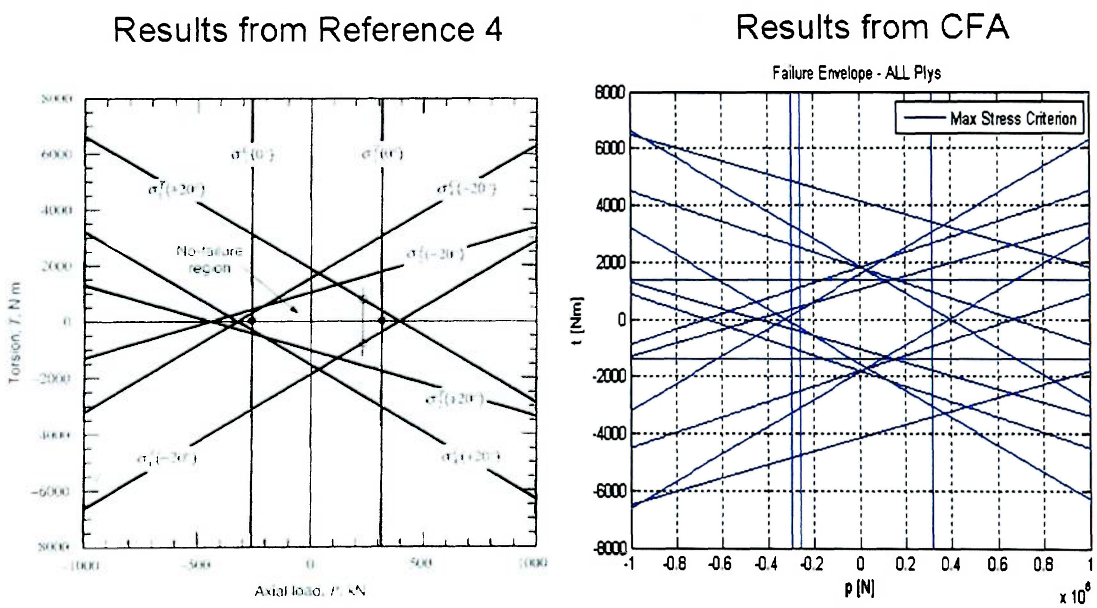

Figure 87: Case 6 – Comparison of Maximum Stress Criterion graphs.

The analysis presented in this section is used as substantiation for the graphical techniques used in the CFA software.

C.7 Case 7: Max Stress Criterion (Laminate w/ N_x) Numerical Validation

This numerical validation case is intended to corroborate CFA's Maximum Stress Criterion calculations for a laminate subjected to an N_x load with that provided from *Reference 4.*

A $[\pm 30/0]_S$ laminate is subjected to force and moment resultants of [4]

$$
\begin{aligned}
N_x &= 102400 N/m \\
N_y &= 0 \\
N_{xy} &= 0 \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned}
\tag{c.15}
$$

The summary of loads N_x (MN/m) to cause failure based upon the Maximum Stress Criterion is given in Figure 88, from the reference example.

Layer	Failure mode					
	σ_1^C	σ_1^T	σ_2^C	σ_2^T	$-\tau_{12}^F$	$+\tau_{12}^F$
+30	1 444	+1 733	+2 59	−0 647	+0 973	0 973
−30	1 444	+1 733	+2 59	0 647	−0 973	+0 973
0	0 659	+0 791	+1 439	0 360	∞	+∞

Figure 88: Case 7 – Reference loads to cause failure [4].

From CFA, the equivalent summary of loads to cause failure based upon the Maximum Stress Criterion is given in Figure 89.

Layer	Loads Nx [N/m] To Cause Failure For Specified Failure Mode					
	σ_1^T	σ_1^C	σ_2^T	σ_2^C	τ_{12}^F	$-\tau_{12}^F$
1	1732603 7	-1443836	-646765 3	2587061 3	-973246 3	973246 26
2	1732603 7	-1443836	-646765 3	2587061 3	973246 26	-973246 3
3	791392 88	-659494 1	-359863 6	1439454 5	∞	-∞
4	791392 88	-659494 1	-359863 6	1439454 5	∞	-∞
5	1732603 7	-1443836	-646765 3	2587061 3	973246 26	-973246 3
6	1732603 7	-1443836	-646765 3	2587061 3	-973246 3	973246 26

Figure 89: Case 7 – CFA's calculated loads to cause extensional laminate failure.

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.8 Case 8: Tsai-Wu Criterion (Axially Loaded Tube) Numerical Validation

This numerical validation case is intended to corroborate CFA's Tsai-Wu Criterion calculations of an axially loaded tube with that provided from *Reference 4.*

A $[\pm 20/0_3]_S$ laminate is subjected to force and moment resultants of [4]

$$
\begin{aligned}
N_x &= 6.37 N/m \\
N_y &= 0 \\
N_{xy} &= 0 \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned}
\tag{c.16}
$$

The summary of loads P (MN) to cause failure based upon the Tsai-Wu Criterion is given in Figure 90, from the reference example.

	+20° layers		−20° layers		0° layers	
	$P = -198$	$P = +350$	$P = -198$	$P = +350$	$P = -156.0$	$P = +308$
σ_1, MPa	-758	1340	-758	1340	743	1467
σ_2, MPa	22.2	39.3	22.2	-39.3	26.3	51.9
τ_{12}, MPa	29.4	52.1	29.4	52.1	0	0
$F_1\sigma_1$	0.101	-0.179	0.101	-0.179	0.099	-0.196
$F_2\sigma_2$	0.334	0.590	0.334	-0.590	0.394	-0.779
$F_{11}\sigma_1^2$	0.306	0.958	0.306	0.958	0.295	1.149
$F_{22}\sigma_2^2$	0.049	0.155	0.049	0.155	0.069	0.269
$F_{66}\tau_{12}^2$	0.087	0.271	0.087	0.271	0	0
$-\sqrt{F_{11}F_{22}}\,\sigma_1\sigma_2$	0.123	0.385	0.123	0.385	0.113	0.556
Total	1.000	1.000	1.000	1.000	1.000	1.000

Figure 90: Case 8 - Reference loads to cause failure [4].

From CFA, the equivalent summary of loads to cause failure based upon the Tsai-Wu Criterion is given in Figure 91, Figure 92, and Figure 93.

	Failure Load P=p [N]	
Layer	p (neg)	p (pos)
1	-197878	350004 2
2	-197878	350004 2
3	-155869	307728 3
4	-155869	307728 3
5	-155869	307728 3
6	-155869	307728 3
7	-155869	307728 3
8	-155869	307728 3
9	-197878	350004 2
10	-197878	350004 2

Figure 91: Case 8 – CFA's calculated loads to cause axial laminate failure (1/3).

Layer	Summary for Negative Load P=p (neg) [N] to Cause Failure									
	σ_1 (Pa)	σ_2 (Pa)	τ_{12} (Pa)	$F_1\sigma_1$	$F_2\sigma_2$	$F_{11}\sigma_1^2$	$F_{22}\sigma_2^2$	$F_{66}\tau_{12}^2$	$-\sqrt{(F_{11}F_{11})}\sigma_1\sigma_2$	Total
1	-7 6E+08	22241375	29440672	0 10102	0 333621	0 306152	0 049468	0 086675	0 123063804	1
2	-7 6E+08	22241375	-2 9E+07	0 10102	0 333621	0 306152	0 049468	0 086675	0 123063804	1
3	-7 4E+08	26291748	0	0 099108	0 394376	0 29467	0 069126	0	0 142720789	1
4	-7 4E+08	26291748	0	0 099108	0 394376	0 29467	0 069126	0	0 142720789	1
5	-7 4E+08	26291748	0	0 099108	0 394376	0 29467	0 069126	0	0 142720789	1
6	-7 4E+08	26291748	0	0 099108	0 394376	0 29467	0 069126	0	0 142720789	1
7	-7 4E+08	26291748	0	0 099108	0 394376	0 29467	0 069126	0	0 142720789	1
8	-7 4E+08	26291748	0	0 099108	0 394376	0 29467	0 069126	0	0 142720789	1
9	-7 6E+08	22241375	-2 9E+07	0 10102	0 333621	0 306152	0 049468	0 086675	0 123063804	1
10	-7 6E+08	22241375	29440672	0 10102	0 333621	0 306152	0 049468	0 086675	0 123063804	1

Figure 92: Case 8 – CFA's calculated loads to cause axial laminate failure (2/3).

Layer	Summary for Positive Load P=p (pos) [N] to Cause Failure									
	σ_1 (Pa)	σ_2 (Pa)	τ_{12} (Pa)	$F_1\sigma_1$	$F_2\sigma_2$	$F_{11}\sigma_1^2$	$F_{22}\sigma_2^2$	$F_{66}\tau_{12}^2$	$-\sqrt{(F_{11}F_{22})}\sigma_1\sigma_2$	Total
1	1 34E+09	-3 9E+07	-5 2E+07	-0 17868	-0 5901	0 95783	0 154766	0 271173	0 38501832	1
2	1 34E+09	-3 9E+07	52074277	-0 17868	-0 5901	0 95783	0 154766	0 271173	0 38501832	1
3	1 47E+09	-5 2E+07	0	-0 19567	-0 77861	1 148548	0 269434	0	0 556289255	1
4	1 47E+09	-5 2E+07	0	-0 19567	-0 77861	1 148548	0 269434	0	0 556289255	1
5	1 47E+09	-5 2E+07	0	-0 19567	-0 77861	1 148548	0 269434	0	0 556289255	1
6	1 47E+09	-5 2E+07	0	-0 19567	-0 77861	1 148548	0 269434	0	0 556289255	1
7	1 47E+09	-5 2E+07	0	-0 19567	-0 77861	1 148548	0 269434	0	0 556289255	1
8	1 47E+09	-5 2E+07	0	-0 19567	-0 77861	1 148548	0 269434	0	0 556289255	1
9	1 34E+09	-3 9E+07	52074277	-0 17868	-0 5901	0 95783	0 154766	0 271173	0 38501832	1
10	1 34E+09	-3 9E+07	-5 2E+07	-0 17868	-0 5901	0 95783	0 154766	0 271173	0 38501832	1

Figure 93: Case 8 – CFA's calculated loads to cause axial laminate failure (3/3).

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.9 Case 9: Tsai-Wu Criterion (Tube in Torsion) Numerical Validation

This numerical validation case is intended to corroborate CFA's Tsai-Wu Criterion calculations of a torsionally loaded tube with that provided from *Reference 4.*

A $[\pm 20/0_3]_S$ laminate is subjected to force and moment resultants of [4]

$$\begin{aligned} N_r &= \frac{P}{2\pi R} = 1432394N/m \\ N_y &= 0 \\ N_{xy} &= \frac{T}{2\pi R^2} = 255N/m \\ M_x &= 0 \\ M_y &= 0 \\ M_{xy} &= 0 \end{aligned} \tag{c.17}$$

The summary of loads T (MNm) to cause failure based upon the Tsai-Wu Criterion is given in Figure 94, from the reference example.

	+20° layers		-20 layers		0° layers	
	T = -917	T = +715	T = -715	T = +917	T = -1125	T = -1125
σ_1, MPa	123.7	1436	1436	123.7	1072	1072
σ_2, MPa	18.86	59.7	59.7	18.86	37.9	-37.9
τ_{12}, MPa	-84.1	6.06	6.06	84.1	-81.1	81.1
$F_1\sigma_1$	-0.016	0.192	-0.192	-0.016	0.143	-0.143
$F_2\sigma_2$	0.283	-0.896	-0.896	0.283	-0.569	-0.569
$F_{11}\sigma_1^2$	0.008	1.100	1.100	0.008	0.613	0.613
$F_{22}\sigma_2^2$	0.036	0.357	0.357	0.036	0.144	0.144
$F_{66}\tau_{12}^2$	0.707	0.004	0.004	0.707	0.655	0.658
$-\sqrt{F_{11}F_{22}}\sigma_1\sigma_2$	-0.017	0.627	0.626	-0.017	0.297	0.297
Total	1.000	1.000	1.000	1.000	1.000	1.000

Figure 94: Case 9 - Reference loads to cause failure [4].

From CFA, the equivalent summary of loads to cause failure based upon the Tsai-Wu Criterion is given in Figure 95 and Figure 96.

	Failure Load P=p [N]	
Layer	p (neg)	p (pos)
1	-915.561	714.5639
2	-714.564	915.5608
3	-1123.37	1123 367
4	-1123.37	1123.367
5	-1123.37	1123.367
6	-1123.37	1123.367
7	-1123.37	1123.367
8	-1123.37	1123.367
9	-714.564	915.5608
10	-915.561	714.5639

Figure 95: Case 9 – CFA's calculated loads to cause torsional laminate failure (1/2).

	Summary for Negative Load P=p (neg) [N] to Cause Failure									
Layer	σ_1 (Pa)	σ_2 (Pa)	τ_{12} (Pa)	$F_1\sigma_1$	$F_2\sigma_2$	$F_{11}\sigma_1^2$	$F_{22}\sigma_2^2$	$F_{66}\tau_{12}^2$	$-\sqrt{(F_{11}F_{11})}\sigma_1\sigma_2$	Total
1	123578056	18851486	-84085392	-0.01648	0.282772	0 008145	0.035538	0.707035	-0.017013212	1
2	1.437E+09	-6E+07	-6058635	-0 19153	-0 89571	1 100555	0 356574	0 003671	0 626441342	1
3	1 072E+09	-3.8E+07	-81101095	-0.14298	-0.56895	0.613282	0 143868	0.657739	0 297038029	1
4	1.072E+09	-3 8E+07	-81101095	-0.14298	-0.56895	0.613282	0 143868	0.657739	0.297038029	1
5	1.072E+09	-3.8E+07	-81101095	-0.14298	-0 56895	0.613282	0.143868	0 657739	0 297038029	1
6	1 072E+09	-3.8E+07	-81101095	-0 14298	-0.56895	0 613282	0 143868	0 657739	0 297038029	1
7	1 072E+09	-3.8E+07	-81101095	-0.14298	-0.56895	0 613282	0 143868	0 657739	0.297038029	1
8	1.072E+09	-3.8E+07	-81101095	-0 14298	-0.56895	0.613282	0 143868	0.657739	0.297038029	1
9	1 437E+09	-6E+07	-6058635	-0.19153	-0.89571	1 100555	0.356574	0.003671	0.626441342	1
10	123578056	18851486	-84085392	-0 01648	0 282772	0 008145	0.035538	0 707035	-0 017013212	1

	Summary for Positive Load P=p (pos) [N] to Cause Failure									
Layer	σ_1 (Pa)	σ_2 (Pa)	τ_{12} (Pa)	$F_1\sigma_1$	$F_2\sigma_2$	$F_{11}\sigma_1^2$	$F_{22}\sigma_2^2$	$F_{66}\tau_{12}^2$	$-\sqrt{(F_{11}F_{22})}\sigma_1\sigma_2$	Total
1	1436502791	-59713781	6058635	-0.19153	-0.89571	1.100555	0.356574	0.003671	0.626441342	1
2	123578056	18851486	84085392	-0.01648	0.282772	0 008145	0.035538	0.707035	-0.017013212	1
3	1072335827	-37929915	81101095	-0.14298	-0.56895	0.613282	0.143868	0.657739	0.297038029	1
4	1072335827	-37929915	81101095	-0 14298	-0.56895	0.613282	0 143868	0.657739	0 297038029	1
5	1072335827	-37929915	81101095	-0.14298	-0.56895	0.613282	0.143868	0.657739	0.297038029	1
6	1072335827	-37929915	81101095	-0.14298	-0.56895	0.613282	0.143868	0.657739	0.297038029	1
7	1072335827	-37929915	81101095	-0.14298	-0.56895	0 613282	0.143868	0.657739	0.297038029	1
8	1072335827	-37929915	81101095	-0.14298	-0.56895	0.613282	0 143868	0.657739	0.297038029	1
9	123578056	18851486	84085392	-0.01648	0.282772	0.008145	0 035538	0.707035	-0 017013212	1
10	1436502791	-59713781	6058635	-0.19153	-0.89571	1.100555	0.356574	0.003671	0.626441342	1

Figure 96: Case 9 – CFA's calculated loads to cause torsional laminate failure (2/2).

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.10 Case 10: Tsai-Wu Criterion (Tube w/ Combined Load) Graphical Validation

This graphical validation case is intended to corroborate CFA's Tsai-Wu Criterion calculations for a tube with a combined axial and torsional loading with that provided from *Reference 4*.

Using a $[\pm 20/0_3]_S$ tube subjected to force and moment resultants of [4]

$$\begin{aligned}
N_x &= \frac{P}{2\pi R} = 6.37N/m \\
N_y &= 0 \\
N_{xy} &= \frac{T}{2\pi R^2} = 255N/m \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned} \tag{c.18}$$

an illustration of their graphical comparison is given in Figure 97.

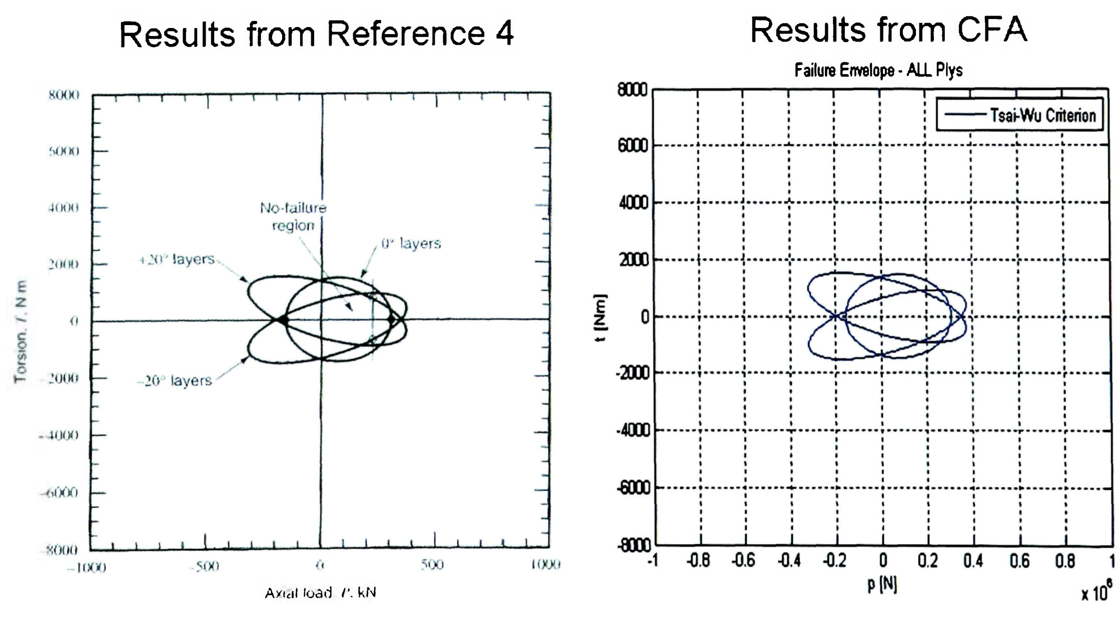

Figure 97: Case 10 – Comparison of Tsai-Wu Criterion graphs.

The analysis presented in this section is used as substantiation for the graphical techniques used in the CFA software.

C.11 Case 11: Tsai-Wu Criterion (Laminate w/ N_x) Numerical Validation

This numerical validation case is intended to corroborate CFA's Tsai-Wu Criterion calculations for a laminate subjected to an N_x load with that provided from *Reference 4.*

A $[\pm 30/0]_S$ laminate is subjected to force and moment resultants of [4]

$$
\begin{aligned}
N_x &= 1 N/m \\
N_y &= 0 \\
N_{xy} &= 0 \\
M_x &= 0 \\
M_y &= 0 \\
M_{xy} &= 0
\end{aligned}
\tag{c.19}
$$

The summary of loads N_x (MN/m) to cause failure based upon the Maximum Stress Criterion is given in Figure 98, from the reference example.

	+30° layers		−30 layers		0° layers	
	$N_x = -0.425$	$N_x = +0.926$	$N_x = -0.425$	$N_x = +0.926$	$N_x = -0.260$	$N_x = +0.665$
σ_1, MPa	−367	802	367	802	−494	1261
σ_2, MPa	32.8	−71.6	32.8	−71.6	36.2	92.5
τ_{12}, MPa	43.6	95.2	−43.6	95.2	0	0
$F_1\sigma_1$	0.049	−0.106	0.049	−0.107	0.066	0.168
$F_2\sigma_2$	0.492	−1.074	0.492	−1.074	0.542	−1.387
$F_{11}\sigma_1^2$	0.072	0.343	0.072	0.343	0.130	0.848
$F_{22}\sigma_2^2$	0.108	0.513	0.108	0.513	0.131	0.855
$F_{66}\tau_{12}^2$	0.190	0.906	0.190	0.906	0	0
$-\sqrt{F_{11}F_{22}}\sigma_1\sigma_2$	0.088	0.419	0.088	0.419	0.130	0.852
Total	1.000	1.000	1.000	1.000	1.000	1.000

Figure 98: Case 11 - Reference loads to cause failure [4].

From CFA, the equivalent summary of loads to cause failure based upon the Tsai-Wu Criterion is given in Figure 99, Figure 100, and Figure 101.

—	Failure Load P=p [N]	
Layer	p (neg)	p (pos)
1	-424701	926311 6
2	-424701	926311 6
3	-260447	665466 5
4	-260447	665466 5
5	-424701	926311 6
6	-424701	926311 6

Figure 99: Case 11 – CFA's calculated loads to cause extensional laminate failure (1/3).

Layer	Summary for Negative Load P=p (neg) [N] to Cause Failure									
	σ_1 (Pa)	σ_2 (Pa)	τ_{12} (Pa)	$F_1\sigma_1$	$F_2\sigma_2$	$F_{11}\sigma_1^2$	$F_{22}\sigma_2^2$	$F_{66}\tau_{12}^2$	$-\sqrt{(F_{11}F_{11})}\sigma_1\sigma_2$	Total
1	-3.7E+08	32832659	43637518	0.049025	0.49249	0.072102	0.107798	0.190423	0.088161755	1
2	-3.7E+08	32832659	-4.4E+07	0.049025	0.49249	0.072102	0.107798	0.190423	0.088161755	1
3	-4.9E+08	36186956	0	0.06582	0.542804	0.129968	0.13095	0	0.130457942	1
4	-4.9E+08	36186956	0	0.06582	0.542804	0.129968	0.13095	0	0.130457942	1
5	-3.7E+08	32832659	-4.4E+07	0.049025	0.49249	0.072102	0.107798	0.190423	0.088161755	1
6	-3.7E+08	32832659	43637518	0.049025	0.49249	0.072102	0.107798	0.190423	0.088161755	1

Figure 100: Case 11 – CFA's calculated loads to cause extensional laminate failure (2/3).

Layer	Summary for Positive Load P=p (pos) [N] to Cause Failure									
	σ_1 (Pa)	σ_2 (Pa)	τ_{12} (Pa)	$F_1\sigma_1$	$F_2\sigma_2$	$F_{11}\sigma_1^2$	$F_{22}\sigma_2^2$	$F_{66}\tau_{12}^2$	$-\sqrt{(F_{11}F_{22})}\sigma_1\sigma_2$	Total
1	8.02E+08	-7.2E+07	-9.5E+07	-0.10693	-1.07417	0.343002	0.512815	0.905876	0.419400441	1
2	8.02E+08	-7.2E+07	95177516	-0.10693	-1.07417	0.343002	0.512815	0.905876	0.419400441	1
3	1.26E+09	-9.2E+07	0	-0.16818	-1.38691	0.848495	0.854902	0	0.851692597	1
4	1.26E+09	-9.2E+07	0	-0.16818	-1.38691	0.848495	0.854902	0	0.851692597	1
5	8.02E+08	-7.2E+07	95177516	-0.10693	-1.07417	0.343002	0.512815	0.905876	0.419400441	1
6	8.02E+08	-7.2E+07	-9.5E+07	-0.10693	-1.07417	0.343002	0.512815	0.905876	0.419400441	1

Figure 101: Case 11 – CFA's calculated loads to cause extensional laminate failure (3/3).

Again, it is important to recognize that CFA does not employ any rounding techniques during its algorithm execution. The analysis presented in this section is used as substantiation for the algorithms used in the CFA software.

C.12 Case 12: MatLab® Code Validation for Invalid User Input Errors

This error checking validation case for the CFA software is intended to illustrate how CFA deals with erroneous user inputs. Since the MatLab® extension of CFA can only be used for 2D graphical illustrations, it is appropriate to prevent the user from running this extension unless the correct 2D dimensions are given. If the user does not comply, an error message, seen in Figure 102, will be displayed.

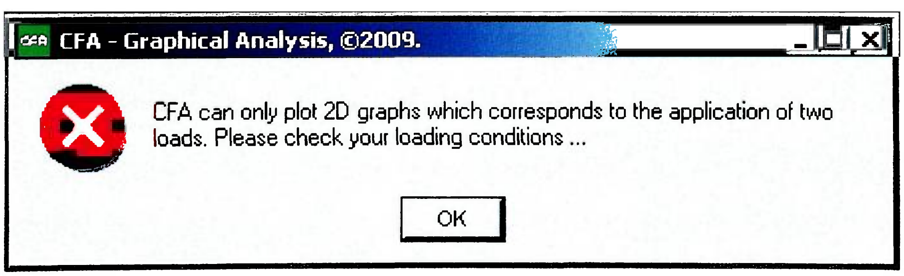

Figure 102: CFA – Error message for incorrect dimensions to graph.

It is necessary to choose a desired failure criterion in order to generate a 2D graph. Therefore, if the user doesn't select a specific failure criterion and tries to generate a graph, an error message, seen in Figure 103, will be displayed.

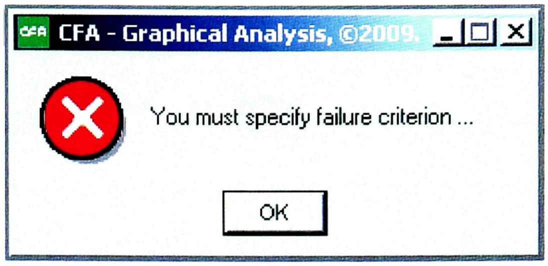

Figure 103: CFA – Error message for no specification of failure criterion.

Lastly, it is necessary to instruct the user that when plotting the piecewise representation of the Tsai-Wu Criterion, more than one layer must be selected to graph. If the user does not comply, an error message, seen in Figure 104, will be displayed.

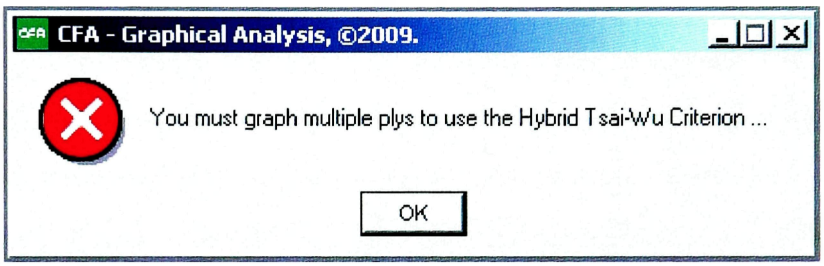

Figure 104: CFA – Error message for not selecting multiple plies to plot.

The analysis presented in this section is used as substantiation for the graphical techniques used in the CFA software.

APPENDIX D

CFA - MatLab® Codes

This appendix provides all of the m-code used to run CFA's MatLab® extension. For functional codes please refer to the attached CFA CD-ROM on the back cover of this thesis document. The CFA MatLab® extension relies heavily on the "xlsread1.m" m-code file in order to import CFA's Excel® based data into MatLab®. This file is included on the attached CFA CD-ROM. However, this code was not written by the author and is available on The MathWorks, Inc.'s online file exchange.

D.1 CFA Graphical User Interface (GUI) Code

```
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 06/04/2009
% File: CFA_Graphical_Analysis.m
%
% Description: This file is used to display a GUI for the user to
% select a specified failure criteria to plot.
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %

function varargout = CFA_Graphical_Analysis(varargin)
%CFA_GRAPHICAL_ANALYSIS M-file for CFA_Graphical_Analysis.fig
%      CFA_GRAPHICAL_ANALYSIS, by itself, creates a new
CFA_GRAPHICAL_ANALYSIS or raises the existing
```

```
%       singleton*.
%
%       H = CFA_GRAPHICAL_ANALYSIS returns the handle to a new
CFA_GRAPHICAL_ANALYSIS or the handle to
%       the existing singleton*.
%
%       CFA_GRAPHICAL_ANALYSIS('Property','Value',...) creates a new
CFA_GRAPHICAL_ANALYSIS using the
%       given property value pairs. Unrecognized properties are passed
via
%       varargin to CFA_Graphical_Analysis_OpeningFcn.  This calling
syntax produces a
%       warning when there is an existing singleton*.
%
%       CFA_GRAPHICAL_ANALYSIS('CALLBACK') and
CFA_GRAPHICAL_ANALYSIS('CALLBACK',hObject,...) call the
%       local function named CALLBACK in CFA_GRAPHICAL_ANALYSIS.M with
the given input
%       arguments.
%
%       *See GUI Options on GUIDE's Tools menu.  Choose "GUI allows only
one
%       instance to run (singleton)".
%
% See also: GUIDE, GUIDATA, GUIHANDLES

% Edit the above text to modify the response to help
CFA_Graphical_Analysis

% Last Modified by GUIDE v2.5 15-May-2009 17:31:53

% Begin initialization code - DO NOT EDIT
gui_Singleton = 1;
gui_State = struct('gui_Name',       mfilename, ...
                   'gui_Singleton',  gui_Singleton, ...
                   'gui_OpeningFcn',
@CFA_Graphical_Analysis_OpeningFcn, ...
                   'gui_OutputFcn',  @CFA_Graphical_Analysis_OutputFcn,
...
                   'gui_LayoutFcn',  [], ...
                   'gui_Callback',   []);
if nargin && ischar(varargin{1})
   gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
% End initialization code - DO NOT EDIT


% --- Executes just before CFA_Graphical_Analysis is made visible.
function CFA_Graphical_Analysis_OpeningFcn(hObject, eventdata, handles,
varargin)
```

```
% This function has no output args, see OutputFcn.
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% varargin   unrecognized PropertyName/PropertyValue pairs from the
%            command line (see VARARGIN)

% Run loading screen.
dlgLD = javax.swing.JDialog();
dpbLD = javax.swing.JProgressBar();
loading_imageicon = javax.swing.ImageIcon('loading screen2.gif');
loading_label = javax.swing.JLabel(loading_imageicon);
current_time = java.lang.System.currentTimeMillis();
goal_time = current_time + 4000;
dlgLD.add(java.awt.BorderLayout.CENTER,dpbLD);
dlgLD.add(java.awt.BorderLayout.NORTH, loading_label);
dlgLD.setDefaultCloseOperation(javax.swing.JDialog.DO_NOTHING_ON_CLOSE)
;
dpbLD.setIndeterminate(true);
dpbLD.setBorder(javax.swing.border.MatteBorder(0,2,2,2,java.awt.Color(.
5,.5,.5)));
dlgLD.setSize(297,111);
dlgLD.setResizable(false);
dlgLD.setAlwaysOnTop(true);
dlgLD.setLocationRelativeTo('');
dlgLD.setUndecorated(true);
dlgLD.validate();
dlgLD.repaint();
dlgLD.show();
while java.lang.System.currentTimeMillis()<=goal_time
end
dlgLD.hide();
dlgLD.dispose();

% Run user license agreement screen.
userDecision = User_Agreement;
if strcmp(userDecision,'Decline')
    error('EXECUTION ERROR: To use CFA, you must accept the terms of
the User License Agreement.');
end

% Choose default command line output for CFA_Graphical_Analysis
handles.output = hObject;

% Define checkbox state variables.
handles.hashinState = 0;
handles.hillState = 0;
handles.hoffmanState = 0;
handles.hybridState = 0;
handles.maxstrainState = 0;
handles.maxstressState = 0;
handles.tsaihillState = 0;
handles.tsaiwuState = 0;
handles.vonmisesState = 0;

% Update handles structure
```

```
guidata(hObject, handles);

% Update GUI title.
set(gcf,'Name',[' CFA - Graphical Analysis, ',char(169),'2009.']);

% UIWAIT makes CFA_Graphical_Analysis wait for user response (see
UIRESUME)
% uiwait(handles.figure1);

% Change figure icon to CFA icon.
warning('off','MATLAB:HandleGraphics:ObsoletedProperty:JavaFrame');
jframe=get(gcf,'javaframe');
jIcon=javax.swing.ImageIcon([pwd,'\CFAicon.png']);
jframe.setFigureIcon(jIcon);


% --- Outputs from this function are returned to the command line.
function varargout = CFA_Graphical_Analysis_OutputFcn(hObject,
eventdata, handles)
% varargout  cell array for returning output args (see VARARGOUT);
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Get default command line output from handles structure
varargout{1} = handles.output;


% --- Executes on button press in maxstress_checkbox.
function maxstress_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to maxstress_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of maxstress_checkbox
handles.maxstressState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in maxstrain_checkbox.
function maxstrain_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to maxstrain_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of maxstrain_checkbox
handles.maxstrainState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);
```

```
% --- Executes on button press in vonmises_checkbox.
function vonmises_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to vonmises_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of vonmises_checkbox
handles.vonmisesState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in hashin_checkbox.
function hashin_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to hashin_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of hashin_checkbox
handles.hashinState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in hill_checkbox.
function hill_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to hill_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of hill_checkbox
handles.hillState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in tsaihill_checkbox.
function tsaihill_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to tsaihill_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of tsaihill_checkbox
handles.tsaihillState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in tsaiwu_checkbox.
function tsaiwu_checkbox_Callback(hObject, eventdata, handles)
```

```
% hObject    handle to tsaiwu_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of tsaiwu_checkbox
handles.tsaiwuState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in hoffman_checkbox.
function hoffman_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to hoffman_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of hoffman_checkbox
handles.hoffmanState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in hybrid_checkbox.
function hybrid_checkbox_Callback(hObject, eventdata, handles)
% hObject    handle to hybrid_checkbox (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Hint: get(hObject,'Value') returns toggle state of hybrid_checkbox
handles.hybridState = get(hObject,'Value');

% Update handles structure
guidata(hObject, handles);


% --- Executes on button press in runanalysis_pushbutton.
function runanalysis_pushbutton_Callback(hObject, eventdata, handles)
% hObject    handle to runanalysis_pushbutton (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
if handles.hashinState || handles.hillState || handles.hoffmanState ||
handles.hybridState || handles.maxstrainState || handles.maxstressState
|| handles.tsaihillState || handles.tsaiwuState ||
handles.vonmisesState
    % Run loading screen.
    dlg = javax.swing.JDialog();
    dpb = javax.swing.JProgressBar();
    loading_imageicon = javax.swing.ImageIcon('loading2.gif');
    loading_label = javax.swing.JLabel(loading_imageicon);
    dlg.add(java.awt.BorderLayout.CENTER, dpb);
    dlg.add(java.awt.BorderLayout.NORTH, loading_label);
```

```
dlg.setDefaultCloseOperation(javax.swing.JDialog.DO_NOTHING_ON_CLOSE);
    dpb.setIndeterminate(true);

dpb.setBorder(javax.swing.border.MatteBorder(0,2,2,2,java.awt.Color(.5,
.5,.5)));
    dlg.setSize(470,248);
    dlg.setResizable(false);
    dlg.setAlwaysOnTop(true);
    dlg.setLocationRelativeTo('');
    dlg.setUndecorated(true);
    dlg.validate();
    dlg.repaint();
    dlg.show();

    % Open and read from Excel.
    Excel = actxserver ('Excel.Application');
    File=[pwd,'\CFA - Composite Failure Analysis.xls'];
    invoke(Excel.Workbooks,'Open',File);
    dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
    totalPlys = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
    plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
    [blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
    if plyNum == 0
        titleString = 'ALL Plys';
    else
        titleString = ['Ply #',num2str(plyNum)];
    end
    Excel.Quit
    Excel.delete
    clear Excel

    if plyNum~=0 && handles.hybridState
        % Display error message.
        msg = msgbox('You must graph multiple plys to use the Hybrid
Tsai-Wu Criterion ...',[' CFA - Graphical Analysis,
',char(169),'2009.'],'error');
        j2frame = get(msg,'javaframe');
        j2Icon=javax.swing.ImageIcon([pwd,'\CFAicon.png']);
        j2frame.setFigureIcon(j2Icon);
        dlg.hide();
        dlg.dispose();
    elseif totalPlys==1 && handles.hybridState
        % Display error message.
        msg = msgbox('You must graph multiple plys to use the Hybrid
Tsai-Wu Criterion ...',[' CFA - Graphical Analysis,
',char(169),'2009.'],'error');
        j2frame = get(msg,'javaframe');
        j2Icon=javax.swing.ImageIcon([pwd,'\CFAicon.png']);
        j2frame.setFigureIcon(j2Icon);
        dlg.hide();
        dlg.dispose();
```

```
    elseif dim~=2
        % Display error message.
        msg = msgbox('CFA can only plot 2D graphs which corresponds to
the application of two loads. Please check your loading conditions
...',[' CFA - Graphical Analysis, ',char(169),'2009.'],'error');
        j2frame = get(msg,'javaframe');
        j2Icon=javax.swing.ImageIcon([pwd,'\CFAicon.png']);
        j2frame.setFigureIcon(j2Icon);
        dlg.hide();
        dlg.dispose();
    else
        % Create plot figure.
        figTitle = [' CFA - Composite Failure Analysis,
',char(169),'2009.'];
        figure('Name',figTitle,'Color','w','NumberTitle','off');
        hold on
        criteriaStrings = {'Hashin Criterion';'Hill Criterion';'Hoffman
Criterion';'Piecewise Tsai-Wu Criterion';'Max Strain Criterion';'Max
Stress Criterion';'Tsai-Hill Criterion';'Tsai-Wu Criterion';'Extended
von Mises Criterion'};
        finalLegend = '';

        % Plot specified failure criteria.
        storeBoolean = zeros(9,1);
        pLocal = 1;
        if handles.hashinState
            setColor(pLocal);
            Hashin_Plot;
            storeBoolean(1) = 1;
            pLocal = pLocal+1;
        end
        if handles.hillState
            setColor(pLocal);
            Hill_Plot;
            storeBoolean(2) = 1;
            pLocal = pLocal+1;
        end
        if handles.hoffmanState
            setColor(pLocal);
            Hoffman_Plot;
            storeBoolean(3) = 1;
            pLocal = pLocal+1;
        end
        if handles.hybridState
            setColor(pLocal);
            Hybrid_Tsai_Wu_Plot;
            storeBoolean(4) = 1;
            pLocal = pLocal+1;
        end
        if handles.maxstrainState
            setColor(pLocal);
            Max_Strain_Plot;
            storeBoolean(5) = 1;
            pLocal = pLocal+1;
        end
        if handles.maxstressState
            setColor(pLocal);
```

```
            Max_Stress_Plot;
            storeBoolean(6) = 1;
            pLocal = pLocal+1;
        end
        if handles.tsaihillState
            setColor(pLocal);
            Tsai_Hill_Plot;
            storeBoolean(7) = 1;
            pLocal = pLocal+1;
        end
        if handles.tsaiwuState
            setColor(pLocal);
            Tsai_Wu_Plot;
            storeBoolean(8) = 1;
            pLocal = pLocal+1;
        end
        if handles.vonmisesState
            setColor(pLocal);
            von_Mises_Plot;
            storeBoolean(9) = 1;
        end

        for j=1:length(storeBoolean)
            if storeBoolean(j)==1
                finalLegend =
cellstr([finalLegend;criteriaStrings(j)]);
            end
        end

        if(strcmp(type,'Tube'))
            xlabel('p [N]');
            ylabel('t [Nm]');
        else
            xlabel('\sigma_1 [Pa]');
            ylabel('\sigma_2 [Pa]');
        end

        [LEGH,OBJH,OUTH,OUTM] =
legend(finalLegend,'Location','NorthEast');
        legend_markers = findobj(OUTH,'type','line');
        qLocal = 1;
        for k = 1:length(storeBoolean)
            if storeBoolean(k)==1
                setLegendColor(legend_markers(qLocal),qLocal);
                qLocal = qLocal+1;
            end
        end

        set(gca,'Title',text('String',['Failure Envelope -
',titleString]));
        grid on
        box on
        j2frame = get(gcf,'javaframe');
        j2Icon=javax.swing.ImageIcon([pwd,'\CFAicon.png']);
        j2frame.setFigureIcon(j2Icon);
        hold off
```

```
        dlg.hide();
        dlg.dispose();
    end
else
    % Display error message.
    msg = msgbox('You must specify failure criterion ...',[' CFA -
Graphical Analysis, ',char(169),'2009.'],'error');
    j2frame = get(msg,'javaframe');
    j2Icon=javax.swing.ImageIcon([pwd,'\CFAicon.png']);
    j2frame.setFigureIcon(j2Icon);
end


% Sets the current plotting color.
function setColor(pValue)
if pValue==1
    set(gca,'ColorOrder',[0 0 1]);%Blue
elseif pValue==2
    set(gca,'ColorOrder',[1 0 0]);%Red
elseif pValue==3
    set(gca,'ColorOrder',[0 .75 .75]);%Cyan
elseif pValue==4
    set(gca,'ColorOrder',[.5 0 1]);%Purple
elseif pValue==5
    set(gca,'ColorOrder',[0 1 0]);%Green
elseif pValue==6
    set(gca,'ColorOrder',[.5 0 .25]);%Dark Red
elseif pValue==7
    set(gca,'ColorOrder',[0 .5 0]);%Dark Green
elseif pValue==8
    set(gca,'ColorOrder',[1 0 .5]);%Pink
elseif pValue==9
    set(gca,'ColorOrder',[1 .5 0]);%Orange
end


% Sets the color for the specified legend entry.
function setLegendColor(handle,qValue)
if qValue==1
    set(handle,'Color',[0 0 1]);%Blue
elseif qValue==2
    set(handle,'Color',[1 0 0]);%Red
elseif qValue==3
    set(handle,'Color',[0 .75 .75]);%Cyan
elseif qValue==4
    set(handle,'Color',[.5 0 1]);%Purple
elseif qValue==5
    set(handle,'Color',[0 1 0]);%Green
elseif qValue==6
    set(handle,'Color',[.5 0 .25]);%Dark Red
elseif qValue==7
    set(handle,'Color',[0 .5 0]);%Dark Green
elseif qValue==8
    set(handle,'Color',[1 0 .5]);%Pink
elseif qValue==9
```

```
    set(handle,'Color',[1 .5 0]);%Orange
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.2 Maximum Stress Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/11/2009
% File: Max_Stress_Plot.m
%
% Description: This file is used to plot the Maximum Stress Criterion
% results from the fiber-reinforced composite failure failure analysis
% that was performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Limit loaded data range.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
CC1t = ['AA7:AA',int2str(m)];
CC1c = ['AB7:AB',int2str(m)];
```

```
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
CC2t = ['AC7:AC',int2str(m)];
CC2c = ['AD7:AD',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
CC3t = ['AE7:AE',int2str(m)];
CC3c = ['AF7:AF',int2str(m)];

% Read data from Excel.
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
C1t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC1t);
C1c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC1c);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
C2t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC2t);
C2c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC2c);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);
C3t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC3t);
C3c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC3c);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
for j=plyNum_lowerbound:plyNum_upperbound
    for i=1:3
        if i==1
            A = A1(j);
            B = B1(j);
            Ct = C1t(j);
            Cc = C1c(j);
```

```
        elseif i==2
            A = A2(j);
            B = B2(j);
            Ct = C2t(j);
            Cc = C2c(j);
        elseif i==3
            A = A3(j);
            B = B3(j);
            Ct = C3t(j);
            Cc = C3c(j);
        end

        if(B==0)
            % Plot tensile function.
            y = limit_y(1):1000:limit_y(2);
            x = (Ct-0*y)/A;
            plot(x,y);
            % Plot compressive function.
            y = limit_y(1):1000:limit_y(2);
            x = (Cc-0*y)/A;
            plot(x,y);
        else
            % Plot tensile function.
            x = limit_x(1):10000:limit_x(2);
            y = (Ct-A*x)/B;
            plot(x,y);
            % Plot compressive function.
            x = limit_x(1):10000:limit_x(2);
            y = (Cc-A*x)/B;
            plot(x,y);
        end
    end
end

% if(strcmp(type,'Tube'))
%     xlabel('p [N]');
%     ylabel('t [N]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end

%set(gca,'Title',text('String',['Maximum Stress Criterion Failure
Envelope - ',titleString]));
%grid on
%box on
%hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.3 Maximum Strain Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/28/2009
```

```
% File: Max_Strain_Plot.m
%
% Description: This file is used to plot the Maximum Strain Criterion
% results from the fiber-reinforced composite failure failure analysis
% that was performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Limit loaded data range.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
CC1t = ['DV7:DV',int2str(m)];
CC1c = ['DW7:DW',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
CC2t = ['DX7:DX',int2str(m)];
CC2c = ['DY7:DY',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
CC3t = ['DZ7:DZ',int2str(m)];
CC3c = ['EA7:EA',int2str(m)];
EE1 = ['DR7:DR',int2str(m)];
EE2 = ['DS7:DS',int2str(m)];
GG12 = ['DT7:DT',int2str(m)];
```

```
VV12 = ['DU7:DU',int2str(m)];

% Read data from Excel.
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
C1t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC1t);
C1c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC1c);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
C2t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC2t);
C2c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC2c);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);
C3t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC3t);
C3c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',CC3c);
E1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',EE1);
E2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',EE2);
G12 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',GG12);
v12 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',VV12);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
for j=plyNum_lowerbound:plyNum_upperbound
    % Plot tensile functions.
    A = (A1(j)/E1(j))-((A2(j)*v12(j))/E1(j));
    B = (B1(j)/E1(j))-((B2(j)*v12(j))/E1(j));
    Ct = C1t(j);
    f = strcat(num2str(A),'*x+',num2str(B),'*y-',num2str(Ct));
```

```
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    A = (A2(j)/E2(j))-((A1(j)*v12(j))/E2(j));
    B = (B2(j)/E2(j))-((B1(j)*v12(j))/E2(j));
    Ct = C2t(j);
    f = strcat(num2str(A),'*x+',num2str(B),'*y-',num2str(Ct));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    A = A3(j);
    B = B3(j);
    Ct = G12(j)*C3t(j);
    f = strcat(num2str(A),'*x+',num2str(B),'*y-',num2str(Ct));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    % Plot compressive functions.
    A = (A1(j)/E1(j))-((A2(j)*v12(j))/E1(j));
    B = (B1(j)/E1(j))-((B2(j)*v12(j))/E1(j));
    Cc = C1c(j);
    f = strcat(num2str(A),'*x+',num2str(B),'*y-',num2str(Cc));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    A = (A2(j)/E2(j))-((A1(j)*v12(j))/E2(j));
    B = (B2(j)/E2(j))-((B1(j)*v12(j))/E2(j));
    Cc = C2c(j);
    f = strcat(num2str(A),'*x+',num2str(B),'*y-',num2str(Cc));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    A = A3(j);
    B = B3(j);
    Cc = G12(j)*C3c(j);
    f = strcat(num2str(A),'*x+',num2str(B),'*y-',num2str(Cc));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [N]');
%     ylabel('t [N]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end
%
% set(gca,'Title',text('String',['Maximum Strain Criterion Failure
Envelope - ',titleString]));
% grid on
% box on
% hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.4 Extended von Mises Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
```

```
% Date: 05/13/2009
% File: von_Mises_Plot.m
%
% Description: This file is used to plot the von Mises Criterion
% results from the fiber-reinforced composite failure analysis that was
% performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
```

```
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);

% Load von Mises coefficient data.
temp1 = ['GO7:GO',int2str(m)];
temp2 = ['GP7:GP',int2str(m)];
temp3 = ['GQ7:GQ',int2str(m)];
temp4 = ['GR7:GR',int2str(m)];
temp5 = ['GT7:GT',int2str(m)];
temp6 = ['GU7:GU',int2str(m)];
temp7 = ['GV7:GV',int2str(m)];
temp8 = ['GW7:GW',int2str(m)];
At = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
Bt = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
Ct = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
Dt = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
Ac = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
Bc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);
Cc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp7);
Dc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp8);

% Load von Mises coefficient data.
temp9 = ['GF7:GF',int2str(m)];
temp10 = ['GG7:GG',int2str(m)];
temp11 = ['GH7:GH',int2str(m)];
s1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp9);
s2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp10);
t12 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp11);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
```

```
for j=plyNum_lowerbound:plyNum_upperbound
    % Calculate von Mises tensile coefficients.
    Ft = At(j)/((s1(j)-s2(j))^2);
    Gt = Bt(j)/(s2(j)^2);
    Ht = Ct(j)/(s1(j)^2);
    Nt = Dt(j)/(t12(j)^2);
    % Determine tensile plot coefficients.
    A = (Ft*(A1(j).^2)) - (2*Ft*A1(j)*A2(j)) + (Ft*(A2(j).^2)) +
(Gt*(A2(j).^2)) + (Ht*(A1(j).^2)) + (Nt*(A3(j).^2));
    B = (Ft*(B1(j).^2)) - (2*Ft*B1(j)*B2(j)) + (Ft*(B2(j).^2)) +
(Gt*(B2(j).^2)) + (Ht*(B1(j).^2)) + (Nt*(B3(j).^2));
    C = (2*Ft*A1(j)*B1(j)) - (2*Ft*A1(j)*B2(j)) - (2*Ft*A2(j)*B1(j)) +
(2*Ft*A2(j)*B2(j)) + (2*Gt*A2(j)*B2(j)) + (2*Ht*A1(j)*B1(j)) +
(2*Nt*A3(j)*B3(j));
    D = -1;
    % Plot tensile function.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    % Calculate von Mises compressive coefficients.
    Fc = Ac(j)/((s1(j)-s2(j))^2);
    Gc = Bc(j)/(s2(j)^2);
    Hc = Cc(j)/(s1(j)^2);
    Nc = Dc(j)/(t12(j)^2);
    % Determine compressive plot coefficients.
    A = (Fc*(A1(j).^2)) - (2*Fc*A1(j)*A2(j)) + (Fc*(A2(j).^2)) +
(Gc*(A2(j).^2)) + (Hc*(A1(j).^2)) + (Nc*(A3(j).^2));
    B = (Fc*(B1(j).^2)) - (2*Fc*B1(j)*B2(j)) + (Fc*(B2(j).^2)) +
(Gc*(B2(j).^2)) + (Hc*(B1(j).^2)) + (Nc*(B3(j).^2));
    C = (2*Fc*A1(j)*B1(j)) - (2*Fc*A1(j)*B2(j)) - (2*Fc*A2(j)*B1(j)) +
(2*Fc*A2(j)*B2(j)) + (2*Gc*A2(j)*B2(j)) + (2*Hc*A1(j)*B1(j)) +
(2*Nc*A3(j)*B3(j));
    D = -1;
    % Plot compressive function.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end
%
% set(gca,'Title',text('String',['von Mises Criterion Failure Envelope
- ',titleString]));
% grid on
% box on
% hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.5 Hashin Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/13/2009
% File: Hashin_Plot.m
%
% Description: This file is used to plot the Hashin Criterion results
%
% from the fiber-reinforced composite failure analysis that was
% performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
```

```
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);

% Load Hashin data.
temp1 = ['HE7:HE',int2str(m)];
temp2 = ['HF7:HF',int2str(m)];
temp3 = ['HG7:HG',int2str(m)];
temp4 = ['HH7:HH',int2str(m)];
temp5 = ['HI7:HI',int2str(m)];
temp6 = ['HJ7:HJ',int2str(m)];
s1t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
s1c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
s2t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
s2c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
t12t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
t12c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
for j=plyNum_lowerbound:plyNum_upperbound
    % Calculate Hashin tensile coefficients.
    F = 1/(s1t(j)^2);
    G = 1/(t12t(j)^2);
    H = 1/(s2t(j)^2);
    % Determine Fiber Tensile Failure plot coefficients.
    A = (F*(A1(j).^2)) + (G*(A3(j).^2));
    B = (F*(B1(j).^2)) + (G*(B3(j).^2));
    C = (2*F*A1(j)*B1(j)) + (2*G*A3(j)*B3(j));
```

```
    D = -1;
    % Plot tensile function for Fiber Tensile Failure.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
    % Determine Matrix Tensile Failure plot coefficients.
    A = (H*(A2(j).^2)) + (G*(A3(j).^2));
    B = (H*(B2(j).^2)) + (G*(B3(j).^2));
    C = (2*H*A2(j)*B2(j)) + (2*G*A3(j)*B3(j));
    D = -1;
    % Plot tensile function for Matrix Tensile Failure.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    % Calculate Hashin compressive coefficients.
    F = 1/(s2c(j)^2);
    G = 1/(t12c(j)^2);
    H = 1/(s1c(j)^2);
    % Determine Matrix Compressive Failure plot coefficients.
    A = (F*(A2(j).^2)) + (G*(A3(j).^2));
    B = (F*(B2(j).^2)) + (G*(B3(j).^2));
    C = (2*F*A2(j)*B2(j)) + (2*G*A3(j)*B3(j));
    D = -1;
    % Plot compressive function for Matrix Compressive Failure.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
    % Determine Fiber-Matrix Shearing Failure plot coefficients.
    A = (H*(A1(j).^2)) + (G*(A3(j).^2));
    B = (H*(B1(j).^2)) + (G*(B3(j).^2));
    C = (2*H*A1(j)*B1(j)) + (2*G*A3(j)*B3(j));
    D = -1;
    % Plot compressive function for Fiber-Matrix Shearing Failure.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    % Calculate Hashin compressive coefficient.
    N = 1/(s1c(j));
    % Determine Fiber Compressive Failure plot coefficients.
    A = N*A1(j);
    B = N*B1(j);
    C = -1;
    % Plot compressive function for Fiber Compressive Failure.
    f = strcat(num2str(A),'*x+',num2str(B),'*y+',num2str(C));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
```

```
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end

%set(gca,'Title',text('String',['Hashin Criterion Failure Envelope - ',titleString]));
%grid on
%box on
%hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.6 Hill Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/13/2009
% File: Hill_Plot.m
%
% Description: This file is used to plot the Hill Criterion results
% from the fiber-reinforced composite failure analysis that was
% performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
```

```
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);

% Load Hill coefficient data.
temp1 = ['FS7:FS',int2str(m)];
temp2 = ['FT7:FT',int2str(m)];
temp3 = ['FU7:FU',int2str(m)];
temp4 = ['FV7:FV',int2str(m)];
temp5 = ['FX7:FX',int2str(m)];
temp6 = ['FY7:FY',int2str(m)];
temp7 = ['FZ7:FZ',int2str(m)];
temp8 = ['GA7:GA',int2str(m)];
Ft = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
Gt = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
Ht = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
Nt = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
Fc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
Gc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);
Hc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp7);
Nc = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp8);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
```

```
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scrsz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
for j=plyNum_lowerbound:plyNum_upperbound
    % Determine tensile plot coefficients.
    A = (Ft(j)*(A2(j).^2)) + (Gt(j)*(A1(j).^2)) + (Ht(j)*(A1(j).^2)) -
(2*Ht(j)*A1(j)*A2(j)) + (Ht(j)*(A2(j).^2)) + (2*Nt(j)*(A3(j).^2));
    B = (Ft(j)*(B2(j).^2)) + (Gt(j)*(B1(j).^2)) + (Ht(j)*(B1(j).^2)) -
(2*Ht(j)*B1(j)*B2(j)) + (Ht(j)*(B2(j).^2)) + (2*Nt(j)*(B3(j).^2));
    C = (2*Ft(j)*A2(j)*B2(j)) + (2*Gt(j)*A1(j)*B1(j)) +
(2*Ht(j)*A1(j)*B1(j)) - (2*Ht(j)*A1(j)*B2(j)) - (2*Ht(j)*A2(j)*B1(j)) +
(2*Ht(j)*A2(j)*B2(j)) + (4*Nt(j)*A3(j)*B3(j));
    D = -1;
    % Plot tensile function.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    % Determine compressive plot coefficients.
    A = (Fc(j)*(A2(j).^2)) + (Gc(j)*(A1(j).^2)) + (Hc(j)*(A1(j).^2)) -
(2*Hc(j)*A1(j)*A2(j)) + (Hc(j)*(A2(j).^2)) + (2*Nc(j)*(A3(j).^2));
    B = (Fc(j)*(B2(j).^2)) + (Gc(j)*(B1(j).^2)) + (Hc(j)*(B1(j).^2)) -
(2*Hc(j)*B1(j)*B2(j)) + (Hc(j)*(B2(j).^2)) + (2*Nc(j)*(B3(j).^2));
    C = (2*Fc(j)*A2(j)*B2(j)) + (2*Gc(j)*A1(j)*B1(j)) +
(2*Hc(j)*A1(j)*B1(j)) - (2*Hc(j)*A1(j)*B2(j)) - (2*Hc(j)*A2(j)*B1(j)) +
(2*Hc(j)*A2(j)*B2(j)) + (4*Nc(j)*A3(j)*B3(j));
    D = -1;
    % Plot compressive function.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end

%set(gca,'Title',text('String',['Hill Criterion Failure Envelope -
',titleString]));
%grid on
%box on
%hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.7 Tsai-Hill Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/12/2009
% File: Tsai_Hill_Plot.m
%
% Description: This file is used to plot the Tsai-Hill Criterion
% results from the fiber-reinforced composite failure analysis that was
% performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
```

```
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);

% Load Tsai-Hill coefficient data.
temp1 = ['FA7:FA',int2str(m)];
temp2 = ['FB7:FB',int2str(m)];
temp3 = ['FC7:FC',int2str(m)];
temp4 = ['FD7:FD',int2str(m)];
temp5 = ['FE7:FE',int2str(m)];
temp6 = ['FF7:FF',int2str(m)];
F1t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
F2t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
F12t = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
F1c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
F2c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
F12c = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
for j=plyNum_lowerbound:plyNum_upperbound
    % Determine tensile plot coefficients.
    At = (F1t(j)*(A1(j).^2)) + (F2t(j)*(A2(j).^2)) -
(F1t(j)*A1(j)*A2(j)) + (F12t(j)*(A3(j).^2));
    Bt = (F1t(j)*(B1(j).^2)) + (F2t(j)*(B2(j).^2)) -
(F1t(j)*B1(j)*B2(j)) + (F12t(j)*(B3(j).^2));
    Ct = (2*F1t(j)*A1(j)*B1(j)) + (2*F2t(j)*A2(j)*B2(j)) -
(F1t(j)*A1(j)*B2(j)) - (F1t(j)*A2(j)*B1(j)) + (2*F12t(j)*A3(j)*B3(j));
    Dt = -1;
    % Plot tensile function.
```

```
    ft =
strcat(num2str(At),'*x^2+',num2str(Bt),'*y^2+',num2str(Ct),'*x*y+',num2
str(Dt));
    ezplot(ft,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);

    % Determine compressive plot coefficients.
    Ac = (F1c(j)*(A1(j).^2)) + (F2c(j)*(A2(j).^2)) -
(F1c(j)*A1(j)*A2(j)) + (F12c(j)*(A3(j).^2));
    Bc = (F1c(j)*(B1(j).^2)) + (F2c(j)*(B2(j).^2)) -
(F1c(j)*B1(j)*B2(j)) + (F12c(j)*(B3(j).^2));
    Cc = (2*F1c(j)*A1(j)*B1(j)) + (2*F2c(j)*A2(j)*B2(j)) -
(F1c(j)*A1(j)*B2(j)) - (F1c(j)*A2(j)*B1(j)) + (2*F12c(j)*A3(j)*B3(j));
    Dc = -1;
    % Plot compressive function.
    fc =
strcat(num2str(Ac),'*x^2+',num2str(Bc),'*y^2+',num2str(Cc),'*x*y+',num2
str(Dc));
    ezplot(fc,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end

%set(gca,'Title',text('String',['Tsai-Hill Criterion Failure Envelope -
',titleString]));
%grid on
%box on
%hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.8 Tsai-Wu Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/12/2009
% File: Tsai_Wu_Plot.m
%
% Description: This file is used to plot the Tsai-Wu Criterion results
% from the fiber-reinforced composite failure analysis that was
% performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
```

```
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank1, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
ANG = ['K7:K',int2str(m)];
MAT = ['G7:G',int2str(m)];
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);
plyAngles = xlsread1('CFA - Composite Failure
Analysis.xls','LAMINATE',ANG);
[blank2, plyMaterials] = xlsread1('CFA - Composite Failure
Analysis.xls','LAMINATE',MAT);

% Load Tsai-Wu coefficient data.
temp1 = ['CZ7:CZ',int2str(m)];
temp2 = ['DA7:DA',int2str(m)];
temp3 = ['DB7:DB',int2str(m)];
temp4 = ['DC7:DC',int2str(m)];
```

```
temp5 = ['DD7:DD',int2str(m)];
temp6 = ['DE7:DE',int2str(m)];
F1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
F2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
F11 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
F22 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
F66 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
sqrtF11F22 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

% Discard duplicate plys.
[angles,plys,blank3] = unique(plyAngles,'rows');
A = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
B = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
C = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
D = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
E = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
F = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);

% Graph standard Tsai-Wu plots.
%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

for j=plyNum_lowerbound:plyNum_upperbound
    % Determine plot coefficients.
    A(j) = (F11(j)*(A1(j).^2)) + (F22(j)*(A2(j).^2)) +
(sqrtF11F22(j)*A1(j)*A2(j)) + (F66(j)*(A3(j).^2));
    B(j) = (F11(j)*(B1(j).^2)) + (F22(j)*(B2(j).^2)) +
(sqrtF11F22(j)*B1(j)*B2(j)) + (F66(j)*(B3(j).^2));
    C(j) = (2*F11(j)*A1(j)*B1(j)) + (2*F22(j)*A2(j)*B2(j)) +
(sqrtF11F22(j)*A1(j)*B2(j)) + (sqrtF11F22(j)*A2(j)*B1(j)) +
(2*F66(j)*A3(j)*B3(j));
    D(j) = (F1(j)*A1(j)) + (F2(j)*A2(j));
    E(j) = (F1(j)*B1(j)) + (F2(j)*B2(j));
    F(j) = -1;

    % Plot function.
    %j = plys(k);
    f =
strcat(num2str(A(j)),'*x^2+',num2str(B(j)),'*y^2+',num2str(C(j)),'*x*y+
',num2str(D(j)),'*x+',num2str(E(j)),'*y+',num2str(F(j)));
```

```
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% for k=1:length(A)
%     % Plot function.
%     j = plys(k);
%     f =
strcat(num2str(A(j)),'*x^2+',num2str(B(j)),'*y^2+',num2str(C(j)),'*x*y+
',num2str(D(j)),'*x+',num2str(E(j)),'*y+',num2str(F(j)));
%     ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
% end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end

%set(gca,'Title',text('String',['Tsai-Wu Criterion Failure Envelope -
',titleString]));
%grid on
%box on
%hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.9 Hoffman Criterion Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
% Author: Ryan C. Schmidt
% Date: 05/14/2009
% File: Hoffman_Plot.m
%
% Description: This file is used to plot the Hoffman Criterion results
% from the fiber-reinforced composite failure analysis that was
% performed using CFA.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
dim = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS','I4');
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
```

```
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    titleString = ['Ply #',num2str(plyNum)];
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);

% Load Hoffman coefficient data.
temp1 = ['IB7:IB',int2str(m)];
temp2 = ['IC7:IC',int2str(m)];
temp3 = ['ID7:ID',int2str(m)];
temp4 = ['IE7:IE',int2str(m)];
temp5 = ['IF7:IF',int2str(m)];
temp6 = ['IG7:IG',int2str(m)];
C1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
C2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
C3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
C4 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
C5 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
```

```
C9 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

%figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
%scrsz = get(0,'ScreenSize');
%figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scr
sz(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
%hold on

% Graph functions.
for j=plyNum_lowerbound:plyNum_upperbound
    % Determine plot coefficients.
    A = (C1(j)*(A1(j).^2)) + (C2(j)*(A2(j).^2)) - (C3(j)*A1(j)*A2(j)) +
(C9(j)*(A3(j).^2));
    B = (C1(j)*(B1(j).^2)) + (C2(j)*(B2(j).^2)) - (C3(j)*B1(j)*B2(j)) +
(C9(j)*(B3(j).^2));
    C = (2*C1(j)*A1(j)*B1(j)) + (2*C2(j)*A2(j)*B2(j)) -
(C3(j)*A1(j)*B2(j)) - (C3(j)*A2(j)*B1(j)) + (2*C9(j)*A3(j)*B3(j));
    D = (C4(j)*A1(j))+(C5(j)*A2(j));
    E = (C4(j)*B1(j))+(C5(j)*B2(j));
    F = -1;
    % Plot function.
    f =
strcat(num2str(A),'*x^2+',num2str(B),'*y^2+',num2str(C),'*x*y+',num2str
(D),'*x+',num2str(E),'*y+',num2str(F));
    ezplot(f,[limit_x(1),limit_x(2),limit_y(1),limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end

%set(gca,'Title',text('String',['Hoffman Criterion Failure Envelope -
',titleString]));
%grid on
%box on
%hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

D.10 Piecewise Tsai-Wu Code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% CFA - Composite Failure Analysis, (c)2009.
%
```

```matlab
% Author: Ryan C. Schmidt
% Date: 05/27/2009
% File: Hybrid_Tsai_Wu_Plot.m
%
% Description: This file is used to show the use of a Piecewise
% Representation Hybrid Failure of the Tsai-Wu Failure Criterion.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Open file input stream.
Excel = actxserver ('Excel.Application');
File=[pwd,'\CFA - Composite Failure Analysis.xls'];
invoke(Excel.Workbooks,'Open',File);

% Define initial values.
n = xlsread1('CFA - Composite Failure Analysis.xls','STRESS
DATA','B19');
[blank1, type] = xlsread1('CFA - Composite Failure
Analysis.xls','RESULTS SUMMARY','D17');
limit_x(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C6');
limit_x(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E6');
limit_y(1) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','C7');
limit_y(2) = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E7');
plyNum = xlsread1('CFA - Composite Failure Analysis.xls','GRAPH
ANALYSIS','E9');
if plyNum == 0
    plyNum_lowerbound = 1;
    plyNum_upperbound = n;
    %titleString = 'ALL Plys';
else
    plyNum_lowerbound = plyNum;
    plyNum_upperbound = plyNum;
    %titleString = ['Ply #',num2string(plyNum)];
end
m = 7+n-1;

% Load stress coefficient data.
AA1 = ['AV7:AV',int2str(m)];
BB1 = ['AX7:AX',int2str(m)];
AA2 = ['AZ7:AZ',int2str(m)];
BB2 = ['BB7:BB',int2str(m)];
AA3 = ['BD7:BD',int2str(m)];
BB3 = ['BF7:BF',int2str(m)];
ANG = ['K7:K',int2str(m)];
MAT = ['G7:G',int2str(m)];
A1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA1);
B1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB1);
A2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA2);
B2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB2);
```

```
A3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',AA3);
B3 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',BB3);
plyAngles = xlsread1('CFA - Composite Failure
Analysis.xls','LAMINATE',ANG);
[blank2, plyMaterials] = xlsread1('CFA - Composite Failure
Analysis.xls','LAMINATE',MAT);

% Load Tsai-Wu coefficient data.
temp1 = ['CZ7:CZ',int2str(m)];
temp2 = ['DA7:DA',int2str(m)];
temp3 = ['DB7:DB',int2str(m)];
temp4 = ['DC7:DC',int2str(m)];
temp5 = ['DD7:DD',int2str(m)];
temp6 = ['DE7:DE',int2str(m)];
F1 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp1);
F2 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp2);
F11 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp3);
F22 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp4);
F66 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp5);
sqrtF11F22 = xlsread1('CFA - Composite Failure Analysis.xls','FAILURE
ANALYSIS',temp6);

% Close file input stream.
Excel.Quit
Excel.delete
clear Excel

% Discard duplicate plys.
[angles,plys,blank3] = unique(plyAngles,'rows');
A = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
B = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
C = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
D = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
E = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);
F = zeros((plyNum_upperbound-plyNum_lowerbound)+1,1);

% Graph standard Tsai-Wu plots.
% figTitle = ['CFA - Composite Failure Analysis, ',char(169),'2009.'];
% scrsz = get(0,'ScreenSize');
%
figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scrs
z(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
ylim([limit_y(1) limit_y(2)]);
xlim([limit_x(1) limit_x(2)]);
% hold on

for j=plyNum_lowerbound:plyNum_upperbound
    % Determine plot coefficients.
```

```
    A(j) = (F11(j)*(A1(j).^2)) + (F22(j)*(A2(j).^2)) +
(sqrtF11F22(j)*A1(j)*A2(j)) + (F66(j)*(A3(j).^2));
    B(j) = (F11(j)*(B1(j).^2)) + (F22(j)*(B2(j).^2)) +
(sqrtF11F22(j)*B1(j)*B2(j)) + (F66(j)*(B3(j).^2));
    C(j) = (2*F11(j)*A1(j)*B1(j)) + (2*F22(j)*A2(j)*B2(j)) +
(sqrtF11F22(j)*A1(j)*B2(j)) + (sqrtF11F22(j)*A2(j)*B1(j)) +
(2*F66(j)*A3(j)*B3(j));
    D(j) = (F1(j)*A1(j)) + (F2(j)*A2(j));
    E(j) = (F1(j)*B1(j)) + (F2(j)*B2(j));
    F(j) = -1;
end

% Determine x-range.
xRange(1) = limit_x(1);
xRange(2) = limit_x(2);
fRange(1) = plyNum_lowerbound;
fRange(2) = plyNum_lowerbound;
for j=plyNum_lowerbound:plyNum_upperbound
    % Determine range.
    temp = (-(sqrt((D(j)^2)-(4*A(j)*F(j)))+D(j)))/(2*A(j));
    if abs(temp) < abs(xRange(1))
        xRange(1) = temp;
        fRange(1) = j;
    end
    temp = (sqrt((D(j)^2)-(4*A(j)*F(j)))-D(j))/(2*A(j));
    if abs(temp) < abs(xRange(2))
        xRange(2) = temp;
        fRange(2) = j;
    end
end

% Determine piecewise ranges.
p = 1;
syms x;
%
figure('Name',figTitle,'Color','w','NumberTitle','off','Position',[scrs
z(3)/4 scrsz(4)/4 scrsz(3)/2 scrsz(4)/2]);
% ylim([limit_y(1) limit_y(2)]);
% xlim([limit_x(1) limit_x(2)]);
% hold on
for a=1:length(plys)
    i=plys(a);
    temp1 = (-(sqrt((-(4*A(i)*B(i)-(C(i)^2))*(x^2))-(2*(2*B(i)*D(i)-
C(i)*E(i))*x)-(4*B(i)*F(i))+(E(i)^2))+(C(i)*x)+E(i)))/(2*B(i));
    func1 = simplify(temp1);
    for b=1:length(plys)
        if a<b
            j=plys(b);
            temp2 = (-(sqrt((-(4*A(j)*B(j)-(C(j)^2))*(x^2))-
(2*(2*B(j)*D(j)-C(j)*E(j))*x)-
(4*B(j)*F(j))+(E(j)^2))+(C(j)*x)+E(j)))/(2*B(j));
            func2 = simplify(temp2);
            solution = solve(func1-func2);
            if length(solution)==1
                xPoints(p) = double(solution);
                p = p+1;
```

```
            else
                for k=1:length(solution)
                    xPoints(p) = double(solution(k));
                    p = p+1;
                end
            end
        end
    end
end

xTemp = union(xPoints,xRange);
[X, index] = sort(xTemp);

% Plot hybrid piecewise function.
for a=1:length(X)-1
    x = X(a);
    xH = X(a)+1;
    for b=1:length(plys)
        i = plys(b);
        if b==1
            tempA1 = (-(sqrt((-(4*A(i)*B(i)-(C(i)^2))*(x^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*x)-
(4*B(i)*F(i))+(E(i)^2))+(C(i)*x)+E(i)))/(2*B(i));
            tempA1H = (-(sqrt((-(4*A(i)*B(i)-(C(i)^2))*(xH^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*xH)-
(4*B(i)*F(i))+(E(i)^2))+(C(i)*xH)+E(i)))/(2*B(i));
            p1 = plys(b);
            tempB1 = (sqrt((-(4*A(i)*B(i)-(C(i)^2))*(x^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*x)-(4*B(i)*F(i))+(E(i)^2))-(C(i)*x)-
E(i))/(2*B(i));
            tempB1H = (sqrt((-(4*A(i)*B(i)-(C(i)^2))*(xH^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*xH)-(4*B(i)*F(i))+(E(i)^2))-(C(i)*xH)-
E(i))/(2*B(i));
            p2 = plys(b);
        else
            tempA2 = (-(sqrt((-(4*A(i)*B(i)-(C(i)^2))*(x^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*x)-
(4*B(i)*F(i))+(E(i)^2))+(C(i)*x)+E(i)))/(2*B(i));
            tempA2H = (-(sqrt((-(4*A(i)*B(i)-(C(i)^2))*(xH^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*xH)-
(4*B(i)*F(i))+(E(i)^2))+(C(i)*xH)+E(i)))/(2*B(i));
            if abs(tempA2H)<abs(tempA1H)
                tempA1 = tempA2;
                tempA1H = tempA2H;
                p1 = plys(b);
            end
            tempB2 = (sqrt((-(4*A(i)*B(i)-(C(i)^2))*(x^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*x)-(4*B(i)*F(i))+(E(i)^2))-(C(i)*x)-
E(i))/(2*B(i));
            tempB2H = (sqrt((-(4*A(i)*B(i)-(C(i)^2))*(xH^2))-
(2*(2*B(i)*D(i)-C(i)*E(i))*xH)-(4*B(i)*F(i))+(E(i)^2))-(C(i)*xH)-
E(i))/(2*B(i));
            if abs(tempB2H)<abs(tempB1H)
                tempB1 = tempB2;
                tempB1H = tempB2H;
                p2 = plys(b);
```

```
            end
        end
    end
    j = p1;
    k = p2;
    f1 =
strcat(num2str(A(j)),'*x^2+',num2str(B(j)),'*y^2+',num2str(C(j)),'*x*y+
',num2str(D(j)),'*x+',num2str(E(j)),'*y+',num2str(F(j)));
    f2 =
strcat(num2str(A(k)),'*x^2+',num2str(B(k)),'*y^2+',num2str(C(k)),'*x*y+
',num2str(D(k)),'*x+',num2str(E(k)),'*y+',num2str(F(k)));
    ezplot(f1,[X(a),X(a+1),limit_y(1),0]);
    ezplot(f2,[X(a),X(a+1),0,limit_y(2)]);
end

% if(strcmp(type,'Tube'))
%     xlabel('p [Pa]');
%     ylabel('t [Pa]');
% else
%     xlabel('\sigma_1 [Pa]');
%     ylabel('\sigma_2 [Pa]');
% end
%
% set(gca,'Title',text('String',['Hybrid Criterion Failure Envelope -
',titleString]));
% grid on
% box on
% hold off
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

An Intelligible Software (CFA) Approach for Fiber-Reinforced Laminate Failure Analysis Including a Piecewise Representation of the Tsai-Wu Failure Criterion Using Excel® and MatLab®

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